Homework for Lecture 1 of Dr. Z.'s Dynamical Models in Biology class

Email the answers to

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by 8:00pm Monday, Sept. 8, 2025.

Subject: HW

with an attachment LastFirstHw1.pdf

1. Compute the first six terms of the sequence satisfying the recurrence equation

$$x_n = x_{n-1} + 2x_{n-2} - x_{n-3} \quad , \quad n \geq 3$$

$x_3 = x_2 + 2x_1 - x_0 = -1 + 2(2) - 1 = 2$

$x_4 = x_3 + 2x_2 - x_1 = 2 + 2(-1) - 2 = -2$

$x_5 = x_4 + 2x_3 - x_2 = -2 + 2(2) - (-1) = 3$

$x_6 = x_5 + 2x_4 - x_3 = 3 + 2(-2) - 2 = -3$

subject to the initial conditions

x_0	x_1	x_2	x_3	x_4	x_5	x_6
1	2	-1	2	-2	3	-3

$$x_0 = 1, x_1 = 2, x_2 = -1 \quad .$$

2. Solve explicitly the recurrence equation

$$x_n = 5x_{n-1} - 6x_{n-2} \quad ,$$

$0 = x_n - 5x_{n-1} - 6x_{n-2}$

$= r^2 - 5r - 6$

$(r-2)(r-3) \Rightarrow r_1 = 2 \quad r_2 = 3$

$x_n = C_1 2^n + C_2 3^n$

$x_0 = C_1 + C_2 = 0 \rightarrow C_2 = -C_1$

$x_1 = 2C_1 + 3C_2 = 1 \rightarrow 2C_1 + 3(-C_1) \Rightarrow C_1 = 1$

$C_1 = -1$

$-1 + C_2 = 0 \Rightarrow C_2 = 1$

$\boxed{x_n = 3^n - 2^n}$

with initial conditions

$$x_0 = 0, x_1 = 1 \quad .$$

3. (Corrected Sept. 6, 2025, thanks to Caroline Hall [who won a dollar].)

In a certain species of animals, only one-year-old, two-year-old are fertile.

The probabilities of a one-year-old, two-year-old, female to give birth to a new female are p_1, p_2, respectivley.

Assuming that there were c_0 females born at $n = 0$, c_1 females born at $n = 1$ Set up a recurrence that will enable you to find the **expected** number of females born at time n. $\longrightarrow = x_n$

In terms of c_0, c_1, p_1, p_2, how many females were born at $n = 4$?

$x_0 = C_0 \quad x_1 = C_1$

$x_n = p_1 x_{n-1} + p_2 x_{n-2}, n \geq 2$

$x_2 = p_1 x_1 + p_2 x_0 \Rightarrow p_1 C_1 + p_2 C_0$

$x_3 = p_1 x_2 + p_2 x_1 \Rightarrow p_1(p_1 C_1 + p_2 C_0) + p_2 C_1 \Rightarrow (p_1^2 + p_2) C_1 + p_1 p_2 C_0$

$x_4 = p_1 x_3 + p_2 x_2 \Rightarrow p_1((p_1^2 + p_2) C_1 + p_1 p_2 C_0) + p_2(p_1 C_1 + p_2 C_0)$

$\boxed{x_4 = (p_1^3 + p_1 p_2) C_1 + (p_1^2 p_2 + p_2^2) C_0}$