Homework for Lecture 1 of Dr. Z.'s Dynamical Models in Biology class

Email the answers to

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by 8:00pm Monday, Sept. 8, 2025.

Subject: HW

with an attachment LastFirstHw1.pdf

1. Compute the first six terms of the sequence satisfying the recurrence equation

$$x_n = x_{n-1} + 2x_{n-2} - x_{n-3} \quad , \quad n \geq 3$$

subject to the initial conditions

$$x_0 = 1, x_1 = 2, x_2 = -1 \quad .$$

2. Solve explicitly the recurrence equation

$$x_n = 5x_{n-1} - 6x_{n-2} \quad ,$$

with initial conditions

$$x_0 = 0, x_1 = 1 \quad .$$

3. In a certain species of animals, only one-year-old, two-year-old, and three-year-old females are fertile.

The probabilities of a one-year-old, two-year-old, and three-year-old female to give birth to a new female are p_1, p_2, p_3 respectivley.

Assuming that there were c_0 females born at $n = 0$, c_1 females born at $n = 1$, and c_2 females born at $n = 2$. Set up a recurrence that will enable you to find the **expected** number of females born at time n.

In terms of $c_0, c_1, c_2, p_0, p_1, p_2$, how many females were born at $n = 4$?

1. Compute the first six terms of the sequence satisfying the recurrence equation

Idk if you wanted X_0-X_6 or X_3-X_8 so I did more just in case

$$x_n = x_{n-1} + 2x_{n-2} - x_{n-3} \quad , \quad n \geq 3$$

subject to the initial conditions

$$x_0 = 1, x_1 = 2, x_2 = -1 \quad .$$

$$X_3 = X_2 + 2X_1 - X_0$$
$$= -1 + 4 - 1$$
$$= 2$$

$$X_4 = X_3 + 2X_2 - X_1$$
$$= 2 + 2(-1) - 2$$
$$= -2$$

$$X_5 = X_4 + 2(X_3) - X_2$$
$$= -2 + 2(2) + 1$$
$$= 3$$

$$X_6 = X_5 + 2(X_4) - X_3$$
$$= 3 + 2(-2) - 2$$
$$= -3$$

$$X_7 = X_6 + 2(X_5) - X_4$$
$$= -3 + 2(3) + 2$$
$$= 5$$

$$X_8 = X_7 + 2(X_6) - X_5$$
$$= 5 + 2(-3) - 3$$
$$= -4$$

2. Solve explicitly the recurrence equation

$$x_n = 5x_{n-1} - 6x_{n-2} \quad ,$$

with initial conditions

$$x_0 = 0, x_1 = 1 \quad .$$

$$z^2 = 5z^1 - 6z^0$$
$$z^2 - 5z + 6 = 0$$
$$(z-3)(z-2) = 0$$
$$z = 3, 2$$

$$x_n = C_1(3)^n + C_2(2)^n$$

$$X_n = 3^n - 2^n$$

$$X_0 = 0 = C_1 + C_2$$

$$X_1 = 1 = 3C_1 + 2C_2$$
$$1 = 3(-C_2) + 2C_2$$
$$1 = -C_2$$
$$C_2 = -1, C_1 = 1$$

3. In a certain species of animals, only one-year-old, two-year-old, and three-year-old females are fertile.

The probabilities of a one-year-old, two-year-old, and three-year-old female to give birth to a new female are p_1, p_2, p_3 respectivley.

Assuming that there were $\overbrace{c_0 \text{ females born at } n = 0}^{\text{3yr old}}$, $\overbrace{c_1 \text{ females born at } n = 1}^{\text{2yr old}}$, and $\overbrace{c_2 \text{ females born}}^{\text{1yr old}}$ at $n = 2$. Set up a recurrence that will enable you to find the **expected** number of females born at time n.

In terms of $c_0, c_1, c_2, \cancel{p_0, p_1, p_2}$ p_1, p_2, p_3, how many females were born at $n = 4$?

$$C_n = C_{n-1}p_1 + C_{n-2}p_2 + C_{n-3}p_3 \qquad n \geq 3$$

$$C_3 = C_2 p_1 + C_1 p_2 + C_0 p_3$$

$$C_4 = C_3 p_1 + C_2 p_2 + C_1 p_3$$
$$= (C_2 p_1 + C_1 p_2 + C_0 p_3) p_1 + C_2 p_2 + C_1 p_3$$