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Caroline Hill - Homework #1

1. $x_n = x_{n-1} + 2x_{n-2} - x_{n-3}$, $n \geq 3$; $x_0 = 1, x_1 = 2, x_2 = -1$

$$x_3 = x_2 + 2x_1 - x_0 = -1 + 2(2) - (1) = 2$$

$$x_4 = x_3 + 2x_2 - x_1 = 2 + 2(-1) - (2) = -2$$

$$x_5 = x_4 + 2x_3 - x_2 = -2 + 2(2) - (-1) = 3$$

$$x_6 = x_5 + 2x_4 - x_3 = 3 + 2(-2) - (2) = -3$$

$$x_7 = x_6 + 2x_5 - x_4 = -3 + 2(3) - (-2) = 5$$

$$x_8 = x_7 + 2x_6 - x_5 = 5 + 2(-3) - (3) = -4$$

$$x_3 = 2$$

$$x_4 = -2$$

$$x_5 = 3$$

$$x_6 = -3$$

$$x_7 = 5$$

$$x_8 = -4$$

2. $x_n = 5x_{n-1} - 6x_{n-2}$; $x_0 = 0, x_1 = 1$

$$z^2 = 5z - 6; z^2 - 5z + 6 = 0; (z-3)(z-2) = 0; z_1 = 2, z_2 = 3$$

$$\text{gen. sol. } \therefore x_n = C_1(2)^n + C_2(3)^n$$

$$x_0 = C_1 + C_2 = 0; C_1 = -C_2$$

$$x_1 = 2C_1 + 3C_2 = 1$$

$$2(-C_2) + 3C_2 = 1; C_2 = 1 \text{ and } C_1 = -1$$

$$x_n = -(2)^n + 3^n$$

3.

$$n = \text{time in years}; x_0 = c_0, x_1 = c_1, x_2 = c_2$$

start in x_n where $n \geq 3$

$$\text{general recursion: } x_n = p_1 x_{n-1} + p_2 x_{n-2} + p_3 x_{n-3}$$

$$x_3 = p_1 c_2 + p_2 c_1 + p_3 c_0$$

$$x_4 = p_1(p_1 c_2 + p_2 c_1 + p_3 c_0) + p_2 c_2 + p_3 c_1$$

$$x_n = p_1 x_{n-1} + p_2 x_{n-2} + p_3 x_{n-3}$$

$$x_4 = p_1(p_1 c_2 + p_2 c_1 + p_3 c_0) + p_2 c_2 + p_3 c_1$$