Homework for Lecture 1 of Dr. Z.'s Dynamical Models in Biology class

Email the answers to

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by 8:00pm Monday, Sept. 8, 2025.

Subject: HW

with an attachment LastFirstHw1.pdf

1. Compute the first six terms of the sequence satisfying the recurrence equation

$$x_n = x_{n-1} + 2x_{n-2} - x_{n-3}$$
 , $n \ge 3$

subject to the initial conditions

$$x_0 = 1, x_1 = 2, x_2 = -1$$
.

2. Solve explicitly the recurrence equation

$$x_n = 5x_{n-1} - 6x_{n-2} \quad ,$$

with initial conditions

$$x_0 = 0, x_1 = 1$$
.

3. (Corrected Sept. 6, 2025, thanks to Caroline Hall [who won a dollar].)

In a certain species of animals, only one-year-old, two-year-old are fertile.

The probabilities of a one-year-old, two-year-old, female to give birth to a new female are p_1 , p_2 , respectively.

Assuming that there were c_0 females born at n = 0, c_1 females born at n = 1 Set up a recurrence that will enable you to find the **expected** number of females born at time n.

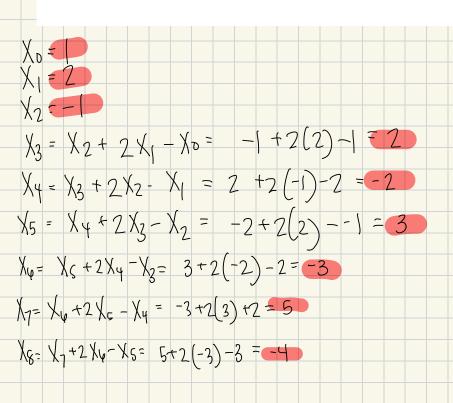
In terms of c_0, c_1, p_1, p_2 , how many females were born at n = 4?

1. Compute the first six terms of the sequence satisfying the recurrence equation

$$x_n = x_{n-1} + 2x_{n-2} - x_{n-3}$$
 , $n \ge 3$

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$$x_0 = 1, x_1 = 2, x_2 = -1$$
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2. Solve explicitly the recurrence equation

$$x_n = 5x_{n-1} - 6x_n 2$$
 $\cancel{\text{order}}$

with initial conditions

$$x_0 = 0, x_1 = 1$$
.

$$Z^{2} - 5Z + (p = 0)$$

$$(Z - 3) (Z - 2) = 0$$

$$Z = 3, 2$$

$$\begin{cases} C_{1} \\ C_{2} \\ C_{3} \end{cases} = 0 = 0$$

$$\begin{cases} C_{1} + C_{2} \\ C_{2} \\ C_{3} \\ C_{4} \end{cases} = 0 = 0$$

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$$\begin{cases} C_{1} + C_{2} \\ C_{2} \end{cases} = 0$$

$$\begin{cases} C_{1} - C_{2} \\ C_{2} \end{cases} = 0$$

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Z2 = 5Z1 - (0Z0

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The probabilities of a one-year-old, two-year-old, female to give birth to a new female are p₁, p₂, respectively.

Assuming that there were c_0 females born at n = 0, c_1 females born at n = 1 Set up a recurrence that will enable you to find the **expected** number of females born at time n.

In terms of c_0, c_1, p_1, p_2 , how many females were born at n = 4?

P1= probability lyrold female gives birth to female p2= probability zyrold female gives birth to female

n=0 -> Co remales born 3 initial cond

cn = females born of time n

Cn = P Cn-3 + P2 Cn-2

C2 = P1 C1 + P2 C0

 $C_3 = P_1 C_2 + P_2 C_1 = P_1 (P_1 C_1 + P_2 C_6) + P_2 C_1$ = $P_1^2 C_1 + P_1 P_2 C_6 + P_2 C_1$

C4= P1 C3+P2C2= P13C1+P12P2C0+P1P2C1+P1P2C1+P22C0

= P 3C1 + 2P P2C1 + P 2P2 C0 + P2 C0