Homework for Lecture 1 of Dr. Z.'s Dynamical Models in Biology class

Email the answers to

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by 8:00pm Monday, Sept. 8, 2025.

Subject: HW

with an attachment LastFirstHw1.pdf

1. Compute the first six terms of the sequence satisfying the recurrence equation

$$x_n = x_{n-1} + 2x_{n-2} - x_{n-3} \quad , \quad n \geq 3$$

subject to the initial conditions

$$x_0 = 1, x_1 = 2, x_2 = -1 \quad .$$

2. Solve explicitly the recurrence equation

$$x_n = 5x_{n-1} - 6x_{n-2} \quad ,$$

with initial conditions

$$x_0 = 0, x_1 = 1 \quad .$$

3. (Corrected Sept. 6, 2025, thanks to Caroline Hall [who won a dollar].)

In a certain species of animals, only one-year-old, two-year-old are fertile.

The probabilities of a one-year-old, two-year-old, female to give birth to a new female are p_1, p_2, respectivley.

Assuming that there were c_0 females born at $n = 0$, c_1 females born at $n = 1$ Set up a recurrence that will enable you to find the **expected** number of females born at time n.

In terms of c_0, c_1, p_1, p_2, how many females were born at $n = 4$?

1. Compute the first six terms of the sequence satisfying the recurrence equation

$$x_n = x_{n-1} + 2x_{n-2} - x_{n-3} \quad , \quad n \geq 3$$

subject to the initial conditions

$$x_0 = 1, x_1 = 2, x_2 = -1 \quad .$$

$X_0 = 1$
$X_1 = 2$
$X_2 = -1$
$X_3 = X_2 + 2X_1 - X_0 = -1 + 2(2) - 1 = 2$
$X_4 = X_3 + 2X_2 - X_1 = 2 + 2(-1) - 2 = -2$
$X_5 = X_4 + 2X_3 - X_2 = -2 + 2(2) - -1 = 3$
$X_6 = X_5 + 2X_4 - X_3 = 3 + 2(-2) - 2 = -3$
$X_7 = X_6 + 2X_5 - X_4 = -3 + 2(3) + 2 = 5$
$X_8 = X_7 + 2X_6 - X_5 = 5 + 2(-3) - 3 = -4$

2. Solve explicitly the recurrence equation

$$x_n = 5x_{n-1} - 6x_{n-2}$$; 2nd order

with initial conditions

$$x_0 = 0, x_1 = 1 \quad .$$

$z^2 = 5z^1 - 6z^0$
$z^2 - 5z + 6 = 0$
$(z-3)(z-2) = 0$
$z = 3, 2$

Gen sol: $X_n = C_1(3)^n + C_2(2)^n$
$X_0 = 0 = C_1 + C_2 \rightarrow C_1 = -C_2$
$X_1 = 1 = 3C_1 + 2C_2$
$-3C_2 + 2C_2 = 1$
$C_2 = -1$
$C_1 = -C_2 = 1$

$\boxed{X_n = 3^n - 2^n}$

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Assuming that there were c_0 females born at $n = 0$, c_1 females born at $n = 1$ Set up a recurrence that will enable you to find the **expected** number of females born at time n.

In terms of c_0, c_1, p_1, p_2, how many females were born at $n = 4$?

p_1 = probability 1yr old female gives birth to female

p_2 = probability 2yr old female gives birth to female

$n=0 \to c_0$ females born
$n=1 \to c_1$ females born } initial cond

c_n = females born at time n

$$c_n = p_1 \underbrace{c_{n-1}}_{\text{1yr old}} + p_2 \underbrace{c_{n-2}}_{\text{2 yr old}}$$

$$c_2 = p_1 c_1 + p_2 c_0$$

$$c_3 = p_1 c_2 + p_2 c_1 = p_1(p_1 c_1 + p_2 c_0) + p_2 c_1$$
$$= p_1^2 c_1 + p_1 p_2 c_0 + p_2 c_1$$

$$c_4 = p_1 c_3 + p_2 c_2 = p_1^3 c_1 + p_1^2 p_2 c_0 + p_1 p_2 c_1 + p_1 p_2 c_1 + p_2^2 c_0$$

$$= p_1^3 c_1 + 2 p_1 p_2 c_1 + p_1^2 p_2 c_0 + p_2^2 c_0$$