Due Sep 8: 800 pm

Homework for Lecture 1 : Ezra Chechik

(1.) Compute the first six terms of the seq. satisfying the recurrence eq.

we need x_0 through x_5, must find sequentially

$x_n = x_{n-1} + 2x_{n-2} - x_{n-3}$, $n \geq 3$

← recurrence

Subject to IC: $x_0 = 1$, $x_1 = 2$, $x_2 = -1$

soln: (* next term to find is x_3, which necessitates x_2, x_1, x_0)

$x_3 = x_2 + 2x_1 - x_0 = (-1) + 2(2) - 1 = 2$

→ $x_3 = 2$ (needed for next computation)

$x_4 = x_3 + 2x_2 - x_1 = 2 + 2(-1) - 2 = -2$

→ $x_4 = -2$

$x_5 = x_4 + 2x_3 - x_2 = (-2) + 2(2) + (-1) = 3$

→ $x_5 = 3$

Final Answer →

First 6 terms: $x_0 = 1, x_1 = 2, x_2 = -1, x_3 = 2, x_4 = -2,$
$x_5 = 3$

(2.) Solve Explicitly: $x_n = 5x_{n-1} - 6x_{n-2}$

IC: $x_0 = 0$, $x_1 = 1$

→ $\dfrac{z^n = 5z^{n-1} - 6z^{n-2}}{z^{n-2}}$ → $z^2 = 5z - 6$

characteristic eq: $z^2 - 5z + 6 = 0$

$(z-3)(z-2) = 0$ two simple roots

$z_1 = 3$, $z_2 = 2$

→ Next Page Continues

2. con't. General Soln: $x_n = c_1(z_1)^n + c_2(z_2)^n$

$\rightarrow x_n = c_1(3)^n + c_2(2)^n$

Determine coefficients. w/ initial conditions.

$x_0 = 0$: $0 = c_1(3)^0 + c_2(2)^0$

$\rightarrow c_1 + c_2 = 0$

System of linear eq.

$x_1 = 1$: $1 = c_1(3)^1 + c_2(2)^1$

$\rightarrow 3c_1 + 2c_2 = 1$

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 3 & 2 & 1 \end{array}\right) \leftrightarrow \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -1 & 1 \end{array}\right) \leftrightarrow \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & -1 \end{array}\right)$$

$$\leftrightarrow \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right) \rightarrow c_1 = 1, \; c_2 = -1$$

$$\boxed{x_n = (3)^n - (2)^n}$$

Expliciit Formula

3. 1 year, 2 year, 3 year xx are Fertile.

$p_1, p_2, p_3 \rightarrow$ probabilities of gen having xx

c_0 Females born at $n=0$, c_1 at $n=1$, c_2 at $n=2$

Setup: $x_n = p_1 x_{n-1} + p_2 x_{n-2} + p_3 x_{n-3}$

In terms of c_n's and p's, how many xx born at $n=4$?

$\longrightarrow$ cont.

$n=3$: $x_3 = p_1 x_2 + p_2 x_1 + p_3 x_0$

$\rightarrow x_3 = p_1 c_2 + p_2 c_1 + p_3 c_0$

$n=4$: $x_4 = p_1 x_3 + p_2 x_2 + p_3 x_1$

$\rightarrow x_4 = p_1(p_1 c_2 + p_2 c_1 + p_3 c_0)$

$+ p_2 c_2 + p_3 c_1$

$\rightarrow \boxed{c_2 p_1^2 + c_1 p_1 p_2 + c_0 p_1 p_3 + c_2 p_2 + c_1 p_3}$

$n=4$, the # of females born given by

Thank You!