Dynamical Models in Biology
HW1

1. $x_n = x_{n-1} + 2x_{n-2} - x_{n-3}$, $n \geq 3$ where $x_0 = 1$, $x_1 = 2$, $x_2 = -1$

Compute:
$$x_3 \Rightarrow (-1) + 2(2) - (1) = 2 = x_3$$
$$x_4 \Rightarrow (2) + 2(-1) - (2) = -2 = x_4$$
$$x_5 \Rightarrow (-2) + 2(2) - (-1) = 3 = x_5$$
$$x_6 \Rightarrow (3) + 2(-2) - (2) = -3 = x_6$$

2. $x_n = 5x_{n-1} - 6x_{n-2}$ where $x_0 = 0$, $x_1 = 1$

Char. Eq.: $z^2 = 5z - 6 \Rightarrow z^2 - 5z + 6 = 0 \Rightarrow (z-2)(z-3)$ so $z = 2, 3 \Rightarrow x_n = C_1(2)^n + C_2(3)^n$

$$x_0 = c_1 + c_2 = 0 \longrightarrow -2c_1 - 2c_2 = 0$$
$$x_1 = 2c_1 + 3c_2 = 1 \qquad 2c_1 + 3c_2 = 1$$
$$c_2 = 1 \longrightarrow c_1 = -1$$

General Solution: $x_n = -(2^n) + 3^n$

3. $p_1 = 1y_0$ gives birth
$p_2 = 2y_0$ gives birth

$$c_n = p_1(c_{n-1}) + p_2(c_{n-2}) \longrightarrow c_2 = p_1c_1 + p_2c_0 \longrightarrow c_3 = p_1(p_1c_1 + p_2c_0) + p_2c_1$$

$$c_4 = \underbrace{p_1(p_1(p_1c_1 + p_2c_0) + p_2c_1)}_{(p_1^2 + p_2)c_1 + p_1p_2c_0} + \underbrace{p_2(p_1c_1 + p_2c_0)}_{p_2p_1c_1 + p_2^2c_0}$$

$$c_4 = (p_1^3 + 2p_1p_2)c_1 + (p_1^2p_2 + p_2^2)c_0$$

Below: The prior version of the Question, which I did prior to the change on 9/6/25

p_1 = prob. of 1 y.o animal giving birth
p_2 = " 2 y.o "
p_3 = " 3 y.o "

If n = years, then p_1 is associated with c_{n-1} in our recurrence, as c_{n-1} is all animals that were born 1 year prior. And so on:

$$C_n = p_1(C_{n-1}) + p_2(C_{n-2}) + p_3(C_{n-3})$$

$$C_3 = p_1c_2 + p_2c_1 + p_3c_0 \rightarrow c_4 = p_1(p_1c_2 + p_2c_1 + p_3c_0) + p_2c_2 + p_3c_1$$

$$c_4 = (p_1p_3)c_0 + (p_1p_2 + p_3)c_1 + (p_1^2 + p_2)c_2$$