Ali Zachariah HW1.pdf

HW1 Dyn Models in Bio

You can post

1.

n	x_n
0	1
1	2
2	-1
3	2
4	-2
5	3

$$x_3 = x_2 + 2x_1 - x_0 = -1 + 2\times 2 - 1 = 2$$
$$x_4 = x_3 + 2x_2 - x_1 = 2 + 2\times -1 - 2 = -2$$
$$x_5 = x_4 + 2x_3 - x_2 = -2 + 2\times 2 - (-1) = 3$$

The first 6 terms are $1, 2, -1, 2, -2, 3$

2. $x_n = 5x_{n-1} - 6x_{n-2}$ $\quad x_0 = 0,\ x_1 = 1$

We will look for a solution of exponential form.
$$x_n = z^n$$
$$\Rightarrow z^n = 5z^{n-1} - 6z^{n-2} \Rightarrow z^2 = 5z - 6$$
$$\Rightarrow z^2 - 5z + 6 = 0 \Rightarrow (z-3)(z-2) = 0 \Rightarrow z_1 = 3 \text{ and } z_2 = 2$$

By the principle of superposition, the general solution has form:-
$$x_n = C_1 z_1^n + \dots + C_k z_k^n$$
So $\quad x_n = C_1 (3)^n + C_2 (2)^n$

with initial conditions $\Rightarrow$ $0 = C_1 \times 1 + C_2 \times 1$ and $1 = C_1 \times 3 + C_2 \times 2$
$$C_1 + C_2 = 0$$
$$3C_1 + 2C_2 = 1$$
$$\Rightarrow C_1 = 1 - 2\times 0 = 1$$
$$\Rightarrow 1 + C_2 = 0$$
so $C_2 = -1$

Thus our explicit solution is $\quad x_n = 3^n - 2^n$

3. We will define E_n to be the expected number of females born at time n.

$E_0 = C_0 \quad E_1 = C_1 \quad E_2 = C_2$

$$E_3 = p_3 C_0 + p_2 C_1 + p_1 C_2$$

The number of 3 year old females multiplied by the probability that one gives birth.

We can use our intuition to say that
$$E_n = p_1 E_{n-1} + p_2 E_{n-2} + p_3 E_{n-3}$$
as we expect there to be E_{n-1} 1 year old females at time n.
The same follows for the other terms.
We know no terms are missing as the animals considered by E_{n-4} are no longer fertile

$$E_3 = p_1 E_2 + p_2 E_1 + p_3 E_0$$
$$= p_1 C_2 + p_2 C_1 + p_3 C_0$$
$$E_4 = p_1 E_3 + p_2 E_2 + p_3 E_1 = p_1(p_1 C_2 + p_2 C_1 + p_3 C_0) + p_2 C_2 + p_3 C_1$$

$$E_4 = c_0 p_1 p_3 + c_1 (p_1 p_2 + p_3) + c_2 (p_1^2 + p_2)$$