HW 9

1]

$$a(n+2) = 5a(n+1) - 6a(n)$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$(\lambda-2)(\lambda-3) \quad \lambda = 2, 3$$

$$a(n) = C_1 2^n + C_2 3^n$$

$$\boxed{a(n) = \left(1 - \frac{2-2^2}{3^2-2^2}\right)2^n + \left(\frac{2-2^2}{3^2-2^2}\right)3^n}$$

$$1 = C_1 + C_2$$

$$2 = C_1 2^2 + C_2 3^2$$

$$2 = (1-C_2)2^2 + C_2 3^2$$

$$2 = 2^2 - C_2 2^2 + C_2 3^2$$

$$\frac{2-2^2}{3^2-2^2} = C_2$$

$$C_1 = 1 - \frac{2-2^2}{3^2-2^2}$$

2] EXACTPD(700, 1000, 0.05)

a) 7/10

b) 210000

c) $6.134 \cdot 10^{-6}$

d) 0.286

3] $a(n) = p\,a(n+1) + (1-p)\,a(n-1)$

$$-p\lambda^2 + \lambda - (1-p) = 0$$

$$(\lambda-1)(p\lambda + p - 1) = 0$$

$$\lambda = 1, \frac{1-p}{p}$$

$$a(n) = A_1 1^n + A_2\left(\frac{1-p}{p}\right)^n$$

$a(0) = 0$, $a(L) = 0$

$$A_1 + A_2 = 0$$

$$A_1 + A_2\left(\frac{1-p}{p}\right) = 1$$

$$-A_2 + A_2\left(\frac{1-p}{p}\right)^L = 1$$

$$\boxed{a_1(n) = \frac{-1}{1-\left(\frac{1-p}{p}\right)^L} + \frac{1}{1-\left(\frac{1-p}{p}\right)^L}\left(\frac{1-p}{p}\right)^n}$$

$$A_2 = \frac{1}{\left(-1 + \left(\frac{1-p}{p}\right)^L\right)}$$

$$A_1 = -\frac{1}{1-\left(\frac{1-p}{p}\right)^L}$$

HW9

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$a(n) = \frac{1}{2}a(n+1) + \frac{1}{2}a(n-1) + 1$ $\quad a(L) = 0$ $\quad a(0) = 0$

$-\frac{1}{2}a(n+1) + a(n) - \frac{1}{2}a(n-1) = 0$

GUESS:

$\lambda^2 - 2\lambda + 1 = 0$ $\quad a(n) = (A_1 n + A_2) n$

$\lambda = 1, 1$

$2(A_1 n^2 + A_2 n) = \frac{1}{2}(A_1(n+1) + A_2)(n+1) + \frac{1}{2}(A_1(n-1) + A_2)(n-1) + 2$

$(A_1 n + A_1 + A_2)(n+1) \quad (A_1 n - A_1 + A_2)(n-1)$

$2A_1 n^2 + 2A_2 n = A_1 n^2 + A_1 n + A_2 n + A_1 n + A_1 + A_2 + A_1 n^2 - A_1 n + A_2 n - A_1 + A_2 + 2$

$2A_2 n = 2A_2 n$ $\quad 2A_1 + 2 = 0$

$A_1 = -1$

$a(n) = (n + A_2) n$

$a(n) = 0 = (L + A_2) L$

$A_2 = L$

$\boxed{a(n) = n(L-n)}$