

MWS

$$6a(n-1) + a(n+3) + 5a(n+1) = 0$$

1)

$$6a(n) + a(n+4) + 5a(n+2) = 0$$

a)

$$a(n+4) = \overset{L_1}{0} a(n+3) + \overset{L_2}{-5} a(n+2) + \overset{L_3}{0} a(n+1) + \overset{L_4}{-6} a(n)$$

b)  $a(n) = \begin{bmatrix} a(n+3) \\ a(n+2) \\ a(n+1) \\ a(n) \end{bmatrix}$

$$A = \begin{bmatrix} \overset{L_1}{0} & \overset{L_2}{-5} & \overset{L_3}{0} & \overset{L_4}{-6} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$a(n+4) = \begin{bmatrix} 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} a(n)$$

c)  $a(0) = 0$   
 $a(1) = 2$   
 $a(2) = 3$   
 $a(3) = 4$

i)

| n | a(n)                                         |
|---|----------------------------------------------|
| 0 | 0                                            |
| 1 | 2                                            |
| 2 | 3                                            |
| 3 | 4                                            |
| 4 | $a(4) = 0a(3) - 5a(2) + 0a(1) - 6a(0) = -15$ |
| 5 | $a(5) = 0a(4) - 5a(3) + 0a(2) - 6a(1) = -32$ |

ii)

$$A^2 = \begin{bmatrix} -5 & 0 & -6 & 0 \\ 0 & -5 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$a(n+5) = A^2 \cdot \begin{bmatrix} 4 \\ 3 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -32 \\ -15 \\ 4 \\ 3 \end{bmatrix}$$

$$a(5) = -32$$

HW5

2

|         | $n_0$ | $n_1$ | $n_2$ | $n_3$ | $n_4$ |
|---------|-------|-------|-------|-------|-------|
| $f_0 =$ | 1.5   | 0.9   | 0.5   | 0.3   |       |
| $p_0$   | 0.8   | 0.7   | 0.6   | 0.6   | N/A   |

a)

$$L = \begin{bmatrix} 0 & 1.5 & 0.9 & 0.5 & 0.3 \\ 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0 \end{bmatrix}$$

b)

$$n_0(0) = 100$$

$$n_1(0) = 90$$

$$n_2(0) = 80$$

$$n_3(0) = 70$$

$$n_4(0) = 60$$

$$n(2) = L^2 \cdot n(0)$$

$$L^2 = \begin{bmatrix} 1.2 & 0.63 & 0.3 & 0.18 & 0 \\ 0 & 1.2 & 0.72 & 0.4 & 0.24 \\ 0.56 & 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 0 & 0 \\ 0 & 0 & 0.36 & 0 & 0 \end{bmatrix}$$

$$n(0) = \begin{bmatrix} 100 \\ 90 \\ 80 \\ 70 \\ 60 \end{bmatrix}$$

$$n(2) = \begin{bmatrix} 213.3 \\ 108 \\ 56 \\ 37.8 \\ 28.8 \end{bmatrix}$$

$$n_3(2) \approx 38$$