

HW 11

$$2 \quad a(n+3) = \frac{1 + 2a(n+2) + 3a(n+1) + 4a(n)}{2 + 3a(n+2) + 4a(n+1) + 5a(n)}$$

$$z = \frac{1 + 2z + 3z + 4z}{2 + 3z + 4z + 5z} = \frac{1 + 9z}{2 + 12z} = 2$$

$$z - \frac{1+9z}{2+12z} = 0 \Rightarrow \frac{12z^2 + 2z - 1 - 9z}{2+12z} = 0$$

$$12z^2 - 7z - 1 = 0$$

$$z = \frac{7 \pm \sqrt{49 - 4(12)(-1)}}{12} = \frac{7 \pm \sqrt{97}}{12}$$

$$3 \quad a(n+3) = \frac{1 + 2a(n+2) + 3a(n+1) + 4a(n)}{2 + 3a(n+2) + 4a(n+1) + 5a(n)}$$

$$x(n) = \begin{bmatrix} a(n+2) \\ a(n+1) \\ a(n) \end{bmatrix} \quad x(n+1) = F(x(n))$$

$$(x_1, x_2, x_3) \rightarrow \left( \frac{1 + 2x_1 + 3x_2 + 4x_3}{2 + 3x_1 + 4x_2 + 5x_3}, x_1, x_2, x_3 \right)$$

HW 11

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$$SS \left( \frac{(1-2x)(1-3x)}{r}, x \right)$$

$$x = \frac{r}{12} + \frac{5}{12} + \frac{\sqrt{r^2 + 10r + 1}}{12}, \quad \frac{r}{12} + \frac{5}{12} - \frac{\sqrt{r^2 + 10r + 1}}{12}$$

$$r_0 \approx 3.45$$

~~maximize~~ orb  $\left( \frac{(1-2x)(1-3x)}{r}, x, 0.7, 1.0, 1.1 \right)$

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i)  $r = 2.5$

ii)  $r = 1.9$

iii)  $r = 1.6$