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hw24, 11/29/21

#1: On the attendance quiz, I messed up the 1st question because the phrasing of it didn't make sense to me, but the explanation made sense

$x''''(t)$ $x'''(t)$ $x''(t)$ $x''(t)$

#2 The rate of the rate of the rate of acceleration is 120 m/s^3

should be m/s^5

x

Initial Values $x'(0)=0$ $x''(0)=0$ $x'''(0)=0$ $x''''(0)=0$

$\int dt$ $x^{(5)}(t)=120$

$x^{(4)}(t)=120t+C_1$

$x^4(0)=0 \rightarrow x^4(t)=120t$

$C_1=0$ $\int dt$ $x^3(t)=60t^2+C_2$

$x^3(0)=0 \longrightarrow x^3(t)=60t^2$

$x^2(t)=20t^3+C_3$

$x^2(0)=0,$ $x''(t)=20t^3$ $x'(t)=5t^4+C_4$

$x(0)=0,$ $x'(t)=5t^4$

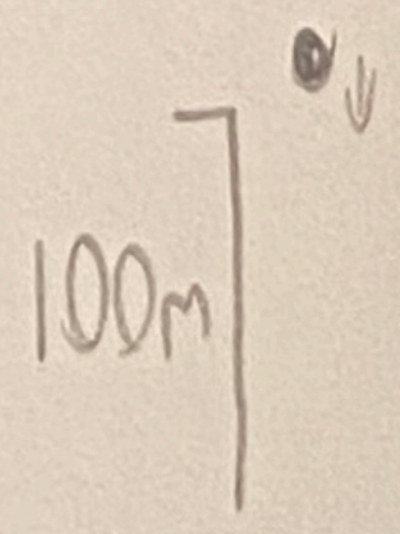
$x(t)=t^5+C_5$

Question: How far from the start is the particle at 2s

$x(t)-C_5 \rightarrow (x-C)=t^5$ at $t=5$, $x-C=32$

The particles distance from start is 32 meters

3i



$$x''(T) = -9.81 \ m/s^2$$

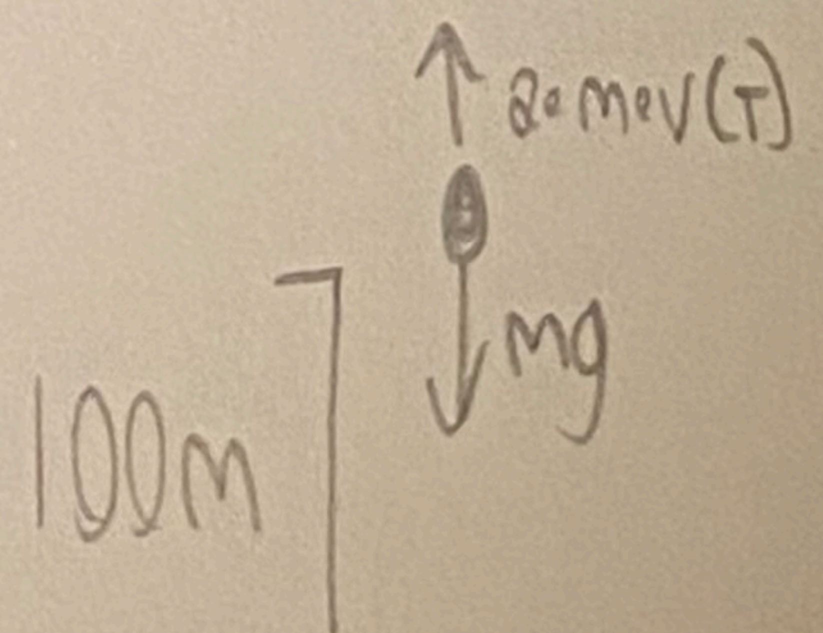
$$x'(T) = -9.81T + C_1 \qquad \text{Initial } x'(0) = 0, \ C_1 = 0$$

$$x(T) = 100 - \frac{9.81}{2}T^2 \leftarrow x(0) = 100$$

$$0 = 100 - 4.905 \, T^2 \qquad \boxed{T = 4.515s}$$

3ii

Air resistance $= 2 \cdot m \cdot v(T) = 2 \cdot m \cdot x'(T)$, opposes Gravity



$$\Sigma F = m \cdot a = m \cdot x''(T)$$

$$\Sigma F = -m \cdot g + 2 \cdot m \cdot x'(T)$$

$$\underline{m} \cdot x''(t) = -\underline{m} \cdot g + 2 \cdot \underline{m} \cdot x'(T)$$

$$x''(T) = -g + 2 \cdot x'(T) = -9.81 + 2 \cdot x'(T)$$

$$x(0) = 100 \qquad x'(0) = 0$$

Applied dsolve in .TXT file

4:

a: A discrete time dynamical system w/ one variable is a system that represents change of a variable over a controlled/discrete time frame

$x(n) = f(x(n-1))$

b: The orbit of a discrete-time dynamical system describes how the variable given (starting with x_0) changes over interval of discrete-time, from 0 to K, by increments of 1.

c: An equilibrium solution is when a given x_0 does not change in an orbit of any amount of discrete time.

Therefore, $x_0 = f(x_0)$ is how to find an equilibrium point

d: A stable equilibrium point has a range of values such that straying from the equilibrium point by those values, the long term behavior of $x(n)$ will return to the equilibrium point.

Found by testing each equilibrium point with $|f'(x_0)|$. If it is less than 1, then it is a stable FP.

5a: To numerically locate stable fixed points of $f(x)$ using the Orb function, one must use random x_0 values and find the orbit of $f(x)$ from x_0 with a long term K, (i.e. 1000) and see if the result is equal to x_0. If it is, and you can increment x_0 by a small value (i.e 0.01) and the orbit still yields x_0, then x_0 is stable.

b: To find the set of equilibrium points, Let $f(x(n-1)) = f(z)$. Then solve $z=f(z)$ for all possible z's. These z values, would equal the set of equilibrium points.

c: To find the subset of b that contains all stable equilibrium points, one must find $f'(z)$, then plug in all values from the set of equilibrium points as z in $f'(z)$. If the absolute value of $f'(z)$ is less than 1, then z is a stable fixed point

5d- All orbits are in the TXT file.

i $x(n) = \dfrac{x(n-1)+1}{x(n-1)+2}$ $f(z) = \dfrac{z+1}{z+2}$

b)

$z = \dfrac{z+1}{z+2} \rightarrow z^2+2z-z-1=0$
$z^2+z-1=0$ $z = \dfrac{-1 \pm \sqrt{1+4}}{2} = -\dfrac{1}{2} \pm \dfrac{\sqrt{5}}{2}$

$FPs = \left\{ \dfrac{-1+\sqrt{5}}{2}, \dfrac{-1-\sqrt{5}}{2} \right\}$

c) $f'(z) = \dfrac{z+2-z-1}{(z+2)^2} = \dfrac{1}{(z+2)^2}$

$\left|f'\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\right| \approx 0.15 < 1$ Stable!

$\left|f'\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\right| \approx 6.85 < 1$ X Not Stable

$SFPs = \left\{ \dfrac{-1+\sqrt{5}}{2} \right\}$

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ii $x(n) = \frac{5}{2} x(n-1) \cdot (1 - x(n-1))$ $f(z) = \frac{5}{2} z(1-z) = \frac{5}{2}z - \frac{5}{2}z^2$

b)

$z = \frac{5}{2}z - \frac{5}{2}z^2 \rightarrow 0 = \frac{3}{2}z - \frac{5}{2}z^2$ $z(3/2 - 5/2 z) = 0$

$z = 0$ & $z = 3/5$

$FPs = \{0, 3/5\}$

c) $f'(z) = 5/2 - 5z$

$|f'(0)| = 5 < 1$ X, not stable!

$|f'(3/5)| = 0.5 < 1$, stable!

$SFPs = \{3/5\}$

iii $x(n) = \frac{7}{2} x(n-1) \cdot (1 - x(n-1))$

b) $f(z) = \frac{7}{2} z \cdot (1-z) = \frac{7}{2} z - \frac{7}{2} z^2$

$z = f(z)$ $\quad z = \frac{7}{2} z - \frac{7}{2} z^2$

$0 = \frac{5}{2} z - \frac{7}{2} z^2$ $\quad z = 0$ & $z = 5/7$

$FP's = \{0, 5/7\}$

c) $f'(z) = 7/2 - 7z$

$|f'(0)| = 7/2 < 1$ X Not stable

$|f'(5/7)| = 3/2 < 1$ X Not stable

$SFP's = \emptyset$