> #OK to post Homework

> #Shreya Ghosh, 11-29-2021, Assignment 24

> **read** "/Users/shreyaghosh/Documents/DMB.txt"

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

>
#3ii)
$dsolve(\{D(D(x))(t) = -10 + 2 \cdot D(x)(t), x(0) = 100, D(x)(0) = 0\}, x(t));$

$$x(t) = -\frac{5\,e^{2t}}{2} + 5t + \frac{205}{2} \quad \textbf{(2)}$$

> $F := -\frac{5\,e^{2t}}{2} + 5t + \frac{205}{2}$

$$F := -\frac{5\,e^{2t}}{2} + 5t + \frac{205}{2} \quad \textbf{(3)}$$

> $fsolve(F)$

$$-20.50000000 \quad \textbf{(4)}$$

> #It will take -20.5 seconds for the ball to reach the ground

>

> #5di)

> $Orb\left(\left[\frac{x+1}{x+2}\right], [x], [0.5], 1000, 1010\right)$

[[0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888]] **(5)**

> #5dii)

> $Orb\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\right], [x], [0.5], 1000, 1010\right)$

[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000]] **(6)**

> #5diii)

> $Orb\left(\left[\frac{7}{2}\cdot x\cdot(1-x)\right], [x], [0.5], 1000, 1010\right)$

[[0.5008842111], [0.8749972637], [0.3828196827], [0.8269407062], [0.5008842111], [0.8749972637], [0.3828196827], [0.8269407062], [0.5008842111], [0.8749972637], [0.3828196827], [0.8269407062]] **(7)**

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Shreya Ghosh, 11/29/2021

HW 24

2. $x''(t) \Rightarrow$ acceleration

$x'''''(t) = 120$, $x'(0)=0$, $x''(0)=0$, $x'''(0)=0$, $x''''(0)=0$

$x''''(t) = 120t + C$

$0 = 0 + C \Rightarrow C = 0$

$x'''(t) = 60t^2 + C$

$0 = 0 + C \Rightarrow C = 0$

$x''(t) = 20t^3 + C$

$0 = 0 + C \Rightarrow C = 0$

$x'(t) = 5t^4 + C$

$0 = 0 + C \Rightarrow C = 0$

$x(t) = t^5 + C$

$0 = 0 + C \Rightarrow C = 0$

$x(2) = 2^5 =$ 32 meters

3. i) $x''(t) = -g$, $x'(0) = 0$, $x(0) = 100$

$x'(t) = -gt + C$

$0 = 0 + C \Rightarrow C = 0$

$x(t) = -\frac{1}{2}gt^2 + C$

$100 = 0 + C \Rightarrow C = 100$

$x(t) = -\frac{1}{2}gt^2 + 100$

$0 = -\frac{1}{2}(10)t^2 + 100$

$0 = -5t^2 + 100$

$20 = t^2$

$\sqrt{20} = t \Rightarrow$ $\sqrt{20}$ sec

3. ii) $\cancel{m}(x''(t)) = -\cancel{m}g + 2\cancel{m}(x'(t))$

$x''(t) = -g + 2x'(t), \quad x'(t) = 0, \quad x(t) = 100$

done in maple

4. a) A discrete dynamical system with one quantity is a dynamical system that changes over discrete time steps and is only dependent upon a single variable within the system.

Format: $x(n) = f(x(n-1))$

b) The orbit of a discrete dynamical system with one quantity starting at $x(n) = x_0$ up to $n=k$ is the collection of points related by the underlying function of the system from time n to K where the initial population is x_0.

c) An equilibrium solution is an instance where the population will not change over time. Can be stable or unstable.

d) A stable equilibrium solution is an instance where the population remains constant over time and the population will always converge back to this point if the initial population starts near this solution.

5. a) With random initial conditions, you would sequence the system using the underlying function until you reach a set of constant points that does not change over time.

b). You would find where the underlying function is equal to the previous population. (i.e. $f(x) = x$)

c). You would take the derivative of the underlying function and see if the absolute value is less than 1 at the fixed points.

di)(b) $x = \frac{x+1}{x+2}$

$x^2 + 2x = x + 1$

$x^2 + x - 1 = 0$

$\frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow x = 0.618034, -1.618034$

(c) $f(x) = \frac{x+1}{x+2}$

$f'(x) = \frac{1(x+2) - 1(x+1)}{(x+2)^2}$

$= \frac{1}{(x+2)^2}$

$f'(0.618034) = 0.145898 \Rightarrow$ stable

$f'(-1.618034) = 6.8541 \Rightarrow$ unstable

dii) (b) $x = \frac{5}{2}x(1-x)$

$x = \frac{5}{2}x - \frac{5}{2}x^2$

$\frac{5}{2}x^2 - \frac{3}{2}x = 0$

$5x^2 - 3x = 0$

$x(5x-3) = 0 \Rightarrow x = 0, \frac{3}{5}$

(c) $f(x) = \frac{5}{2}x(1-x)$

$f'(x) = \frac{5}{2} - 5x$

$f'(0) = \frac{5}{2} \Rightarrow$ unstable

$f'(\frac{3}{5}) = -\frac{1}{2} \Rightarrow$ stable

diii) (b) $x = \frac{7}{2}x(1-x)$

$x = \frac{7}{2}x - \frac{7}{2}x^2$

$\frac{7}{2}x^2 - \frac{5}{2}x = 0$

$7x^2 - 5x = 0$

$x(7x-5) = 0 \Rightarrow x = 0, \frac{5}{7}$

(c) $f(x) = \frac{7}{2}x(1-x)$

$f'(x) = \frac{7}{2} - 7x$

$f'(0) = \frac{7}{2} \Rightarrow$ unstable

$f'(\frac{5}{7}) = -\frac{3}{2} \Rightarrow$ unstable