HW 24 Donut post

1) a) For a certain particle, moving in a straight line, the rate of change of the acceleration is $6\ m/s^3$. At the very beginning its velo and acc are $0\ m/s$ and $0\ m/s^2$

How far from the starting point is the particle after 10 sec

$x'''(t) = 6$

Thus, $x''(t) = 6t + C$

given $x''(0) = 0$ then,

$0 = 6(0) + C$

$0 = C$

$x'(t) = 3t^2 + C$

$0 = 0 + C$

$x(t) = t^3 + C$

$x(t) = t^3$

$\boxed{x(10) = 10^3 = 1000}$

b) A ball is thrown directly vertically from the ground up, at a speed of 5 m/s

i) Set up a differential equation and initial conditions for $x(t)$, its height from the ground until it goes back to the ground.

$$x''(t) = -g$$

with $x'(0) = 5$, $x(0) = 0$

ii) $x''(t) = -g = -10$

$x'(t) = -10t + C$

$5 = C$

$x(t) = \int -10t + 5$

$x(t) = -5t^2 + 5t$, $C = 0$

$-5t^2 + 5t = 0$

$5t(-t+1) = 0$

$t = 0$, $\boxed{t = 1}$

the ball is back after 1sec

2) $120 \ m/s^2$

$z'''(t) = 120 \quad z''(t) = 0 \quad z'(t) = 0 \quad z(0) = 0$

$z''(t) = 120t$

$z'(t) = 60t^2$ $\boxed{c = 0}$

$z(t) = 20t^3$

$z(2) = 20(2)^3 = 20 \cdot 8 = \boxed{160}$ meters

3) i) 100 height

$z''(t) = -g = -10$

$z'(t) = -10t + C$

$z(t) = -5t^2 + C$

$100 = -5t^2 + C$

$C = 100$

$z(t) = -5t^2 + 100$

$-5t^2 = -100$

$5t^2 = 100$

$t^2 = 20$

$\boxed{t = \sqrt{20}\,s}$

ii) $x''(t) = -mg + 2m\,x'(t)$

$x'(0) = 0 \qquad x(0) = 100$

4) a) Format $x(n) = f(x(n-1))$

b) $x(n) = f(x(K-1))$

c) Eq sol. is taking the underlying function and setting it equal to 0 and solving.

d) a stable eq sol of a discrete-time system is by checking if the derivative of the underlying function is abs < 1

5) a) $x(n) = \dfrac{x(n-1)+1}{x(n-1)+2}$

i) we find the underlying function and set it equal to 0
then find the derivative at each eq pt and see if it is abs < 1

ii) We find fixed pts by finding solutions to the underlying function set $= 0$

iii) We use calculus to take the derivative of the underlying function and find the solutions at each eq pt

b) $x(n) = 5/2\, x(n-1)(1 - x(n-1))$

i) Find the underlying function and set it $=0$ then take the derivative at each eq pt

ii) Find the fixed pts by setting the underlying function $=0$ then solving.

iii) We find the stable fixed pts by taking the derivative at each eq pt and solving to see if it is abs <1

c) $x(n) = 7/2\, x(n-1)(1 - x(n-1))$

i) Find the underly function and set it $=0$ then take the derivative at each eq pt.

ii) Find the fixed pts by setting the underly function $=0$ then solve

iii) Find the stable fixed pts by taking the derivative at each eq pt and solving to see if it is abs <1