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Julian Herman, 29th November 2021, Assignment 24

#1)
For the first problem, I didn't integrate the last time and therefore ended up with 3t^2 as my answer. This was due to a simple mistake of rushing and not being careful with my notation.
I did not have the time to do the second problem.

#3)
*#ii)*
*# Keeping to convention, define the up direction as positive and down direction as negative*
*# Let gravitational acceleration be* $-10.0 \frac{meters}{s^2}$
*# Net Force = Force due to air resistance + Force due to gravity (negative) = (2·m·(-x'(t))) +* $(m \cdot -10)$
$= m \cdot (-2 \cdot x'(t) - 10)$
*# Net Force = Mass ·Acceleration*
*#* $m \cdot (-2 \cdot x'(t) - 10) = m \cdot x''(t)$
*#* $-2 \cdot x'(t) - 10 = x''(t)$
*#* $x''(t) + 2 \cdot x'(t) + 10 = 0$
*# x(0)=100; x'(0)=0*
# when the ball hits the ground: x(t)=0.
*dsolve*( {*diff* (*diff* (*x*(*t*), *t*), *t*) + 2·*diff* (*x*(*t*), *t*) + 10 = 0, *x*(0) = 100, *x*'(0) = 0}, {*x*(*t*)})

$$x(t) = -\frac{5\,\mathrm{e}^{-2t}}{2} - 5t + \frac{205}{2} \quad \textbf{(1)}$$

*evalf* (*subs* (*t* = 0, %))

$$x(0) = 100.0000000 \quad \textbf{(2)}$$

$$evalf\left(solve\left(-\frac{5\,\mathrm{e}^{-2t}}{2} - 5t + \frac{205}{2} = 0, t\right)\right)$$

$$20.50000000, -1.90112928 \quad \textbf{(3)}$$

*# The ball hits the ground after 20.5 seconds! This makes sense because with air resistance, it will take longer for the ball to hit the ground than without air resistance (which we calculated in the previous part of the question (#3i) to be 4.47 seconds).*

#4)
#a)
A discrete time dynamical system with one variable / quantity is a relationship between the current value of the variable and previous value(s) of the variable. It represents the evolution of the variable over discrete steps of time via a difference equation.
Format:
# $x(n) = F(x(n-1), x(n-2), x(n-3), \ldots, x(n-k))$ *where* $1 \leq k \leq n$

For a first order: $x(n) = F(x(n-1))$

#b)
The orbit of a discrete-time dynamical system with one-quantity / variable starting at x(n=0)=x0 up to n = K means plugging in the value x0 into the transformation $F$, as described above, in order to determine x(1) and then plugging x(1) into $F$ in order to determine x(2) and so on...performing this composition operation K times (meaning once x(K) is determined, you stop). The orbit is the list of all these values: [x0, x(1), x(2), ..., x(K)] which, again, is calculated by: [x0, F(x0), F(F(x0)),F(F(F(x0))),...]

#c)
An equilibrium solution is a constant x(n)=c of a particular recurrence / difference equation that when plugged into the transformation $F$, yields the same constant c. Therefore, it is a solution to the difference equation that is constant; it repeats itself once it is reached in the recurrence. More formally: x(n)=c is an equilibrium solution if the underlying transformation F(c)=c.

#d)
A stable equilibrium solution is a solution to a particular recurrence / difference equation (as described above) that when you start the recurrence from a point / value x(0) within a local neighborhood / basin of attraction very close to the solution and run its orbit (as described above) it eventually (in the long term) converges to the same solution. More formally: if x(n)=c is an equilibrium solution, then it is a stable equilibrium solution if the limit as n goes to infinity of x(n) = c when the recurrence starts at some initial value c' not equal to c where: $c - \text{delta} < c' < c + \text{delta}$
$(c - \text{delta}, c + \text{delta})$ being the so called "local neighborhood."

#5)
#a)
In order to numerically locate the stable fixed points (aka stable discrete-equilibria) using orbits, you must run Orb() for many iterations (say K1=1000 and K2=1010) with initial conditions very close to the fixed point to be tested and check if the last few values of the orbit are constant and equal to the fixed point being tested. In general: if they are equal and constant, then it is stable; if not, it is unstable.

#b)
If the underlying function is f(x), in order to find the fixed points using algebra you must set f(x)=x and solve for x. This will provide the values of x where the transformation spits out x again and thus x is a fixed point.

#c)
If the underlying function is f(x), to find the subset of the above set of stable fixed points you must take the derivative of f(x): $f'(x)$. Then for each fixed point you must check if: $|f'(x = a\ fixed\ point)| < 1$. If this condition is true, the fixed point is stable (this has to do with the linearizaiton / taylor series approximation). If the condition is not true, the fixed point is unstable (or semi-stable possibly if the LHS evaluates to be exactly 1).

#d)
**read** *`/Users/julianherman/Documents/Rutgers/Fall 2021/Dynamical Models In Biology/HW/DMB.txt`* :

*First Written: Nov. 2021*

*This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)*

*accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)*

*The most current version is available on WWW at:*

*http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .*

*Please report all bugs to: DoronZeil at gmail dot com .*

*For general help, and a list of the MAIN functions,*

*type "Help();". For specific help type "Help(procedure_name);"*

*------------------------------*

*For a list of the supporting functions type: Help1();*

*For help with any of them type: Help(ProcedureName);*

*------------------------------*

*For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();*

*For help with any of them type: Help(ProcedureName);*

*------------------------------*

*For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();*

*For help with any of them type: Help(ProcedureName);*

*------------------------------* **(4)**

#i)

$Help(Orb)$

*Orb(F,x,x0,K1,K2): Inputs a transformation F in the list of variables x with initial point pt, outputs the trajectory of*

*of the discrete dynamical system (i.e. solutions of the difference equation): x(n)=F(x(n-1)) with x(0)= x0 from n=K1 to n=K2.*

*For the full trajectory (from n=0 to n=K2), use K1=0. Try:*

Orb([5/2*x*(1-x)],[x], [0.5], 1000,1010);

Orb([(1+x+y)/(2+x+y),(6+x+y)/(2+4*x+5*y)],[x,y], [2.,3.], 1000,1010); **(5)**

Using (b) to obtain the fixed points:

$$L := evalf\left(\left[solve\left(x = \frac{x+1}{x+2}, x\right)\right]\right)$$

$$L := [0.6180339880, -1.618033988] \quad \textbf{(6)}$$

Using (c) to obtain the stabled fixed points:

$$f(x) := \frac{x+1}{x+2} :$$

**for** *i* **in** *L* **do**: $print\left(evalb\left(|subs(x = i, f'(x))| < 1\right)\right)$ **od**:

*true*

*false* **(7)**

The above tells us that the first fixed point: x=0.6180339880 is stable; and the second fixed point: x= −1.618033988 is unstable.

Using (a) to confirm numerically which of the fixed points are stable:

$$\left\{op\left(Orb\left(\left[\frac{x+1}{x+2}\right], [x], [0.518], 1000, 1010\right)\right)\right\}$$

$$\{[0.6180339888]\} \quad \textbf{(8)}$$

The above shows that x=0.6180339880 is a stable fixed point because starting the orbit at x=0.518 still brings us back to 0.6180339880.

$$\left\{op\left(Orb\left(\left[\frac{x+1}{x+2}\right], [x], [-1.518], 1000, 1010\right)\right)\right\}$$

$$\{[0.6180339888]\} \quad \textbf{(9)}$$

The above shows that x=−1.618033988 is unstable because starting the orbit near it at -1.518 does NOT bring us back to −1.618033988.

#ii)

Using (b) to obtain the fixed points:

$$L := evalf\left(\left[solve\left(x = \frac{5}{2} \cdot x \cdot (1 - x), x\right)\right]\right)$$

$$L := [0., 0.6000000000] \quad \textbf{(10)}$$

Using (c) to obtain the stabled fixed points:

$$f(x) := \frac{5}{2} \cdot x \cdot (1 - x) :$$

**for** *i* **in** *L* **do**: $print\left(evalb\left(|subs(x = i, f'(x))| < 1\right)\right)$ **od**:

*false*

$$true \quad \textbf{(11)}$$

The above tells us that the first fixed point: x=0.0 is unstable; and the second fixed point: x=0.600 is stable.

Using (a) to confirm numerically which of the fixed points are stable:

$$\left\{op\left(Orb\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\right], [x], [0.01], 1000, 1010\right)\right)\right\}$$

$$\{[0.6000000000]\} \quad \textbf{(12)}$$

The above shows that x=0.0

$$\left\{op\left(Orb\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\right], [x], [0.7], 1000, 1010\right)\right)\right\}$$

$$\{[0.6000000000]\} \quad \textbf{(13)}$$

The above shows that x=0.600

#iii)

Using (b) to obtain the fixed points:

$$L := evalf\left(\left[solve\left(x=\frac{7}{2}\cdot x\cdot(1-x), x\right)\right]\right)$$

$$L := [0., 0.7142857143] \quad \textbf{(14)}$$

Using (c) to obtain the stabled fixed points:

$$f(x) := \frac{7}{2}\cdot x\cdot(1-x) :$$

**for** $i$ **in** $L$ **do**: $print\left(evalb\left(|subs(x=i, f'(x))| < 1\right)\right)$**od**:

$$false$$

$$false \quad \textbf{(15)}$$

The above tells us that both of the fixed points are unstable.

Using (a) to confirm numerically that neither of the fixed points are stable:

$$\left\{op\left(Orb\left(\left[\frac{7}{2}\cdot x\cdot(1-x)\right], [x], [0.01], 1000, 1010\right)\right)\right\}$$

$$\{[0.3828196827], [0.5008842111], [0.8269407062], [0.8749972637]\} \quad \textbf{(16)}$$

The above shows that x=0.0 is unstable because starting the orbit near it at 0.01 brings us to a cyclical orbit of period 4 and not back to the fixed point 0.0.

$$\left\{op\left(Orb\left(\left[\frac{7}{2}\cdot x\cdot(1-x)\right], [x], [0.8], 1000, 1010\right)\right)\right\}$$

$$\{[0.3828196827], [0.5008842111], [0.8269407062], [0.8749972637]\} \quad \textbf{(17)}$$

The above shows that x=0.7142857143 is unstable because starting the orbit near it at 0.8 brings us to a cyclical orbit of period 4 and not back to the fixed point 0.7142857143.

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2.) Let: $x(t)$ = position

$x'(t)$ = velocity

$x''(t)$ = acceleration

Then: $x'''''(t) = 120$

$x''''(t) = \int 120 \, dt$

$x''''(t) = 120t + C_1$

$x''''(0) = 120(0) + C_1 = 0 \Rightarrow \boxed{C_1 = 0}$

$x'''(t) = \int 120t \, dt$

$x'''(t) = 60t^2 + C_2$

$x'''(0) = 60(0)^2 + C_2 = 0 \Rightarrow \boxed{C_2 = 0}$

$x''(t) = \int 60t^2 \, dt$

$x''(t) = 20t^3 + C_3$

$x''(0) = 20(0)^3 + C_3 = 0 \Rightarrow \boxed{C_3 = 0}$

$x'(t) = \int 20t^3 \, dt$

$x'(t) = 5t^4 + C_4$

$x'(0) = 5(0)^4 + C_4 = 0 \Rightarrow \boxed{C_4 = 0}$

$x(t) = \int 5t^4 \, dt$

$x(t) = t^5 + C_5$

$x(0) = 0^5 + C_5 = 0 \Rightarrow \boxed{C_5 = 0}$

$\boxed{x(t) = t^5}$

After 2 seconds: $x(2) = 2^5 = 32$ meters

The particle is 32 meters from the starting point

3) i.) acceleration: $x''(t) = -10 \, \frac{m}{s^2}$ (approximately)

$x(0) = 100 \, m$, $x'(0) = 0 \, \frac{m}{s}$ (initial $v = 0 \, \frac{m}{s}$)

$x'(t) = \int -10 \, dt = -10t + C_1$ $\quad x'(0) = -10(0) + C_1 = 0$

$\boxed{C_1 = 0}$

$x(t) = \int -10t \, dt = -5t^2 + C_2$

$x(0) = -5(0)^2 + C_2 = 100 \Rightarrow \boxed{C_2 = 100}$

$$x(t) = -5t^2 + 100$$

When it hits the ground: $x(t) = 0$ m.

$$x(t) = -5t^2 + 100 = 0$$

$$5t^2 = 100$$

$$t^2 = 20$$

$$t = \sqrt{20} = 2\sqrt{5} \approx 4.47 \text{ seconds}$$