Hrudai Batthni Hw 24

1) I was not present and will solve the problems.

1) $\int x'''(t) = \int 6$ $\quad x''(t) = 6t + C$ $\quad x''(0) = 0 \to C = 0$

$\int x''(t) \int 6t = x'(t) = 3t^2 + C$ $\quad x'(0) = 0$ $\quad C = 0$

$\int x'(t) = \int 3t^2 = x(t) = t^3 + C$ $\quad t = 0 \to C = 0$

$x(t) = t^3$ $\quad t = 10$ $\quad \boxed{x(10) = 1000 \text{ meters}}$

2) i) $x''(t) = -g$ $\quad x''(t) = -10$ $\quad x'(0) = 5$ $\quad x(0) = 0$

ii) $\int x''(t) = \int -g = x'(t) = -gt + C$ $\quad 5 = C$ $\quad \int x'(t) = \int gt + 5$

$x(t) = -\frac{gt^2}{2} + 5t + C$ $\quad C = 0$ $\quad x(t) = -\frac{gt^2}{2} + 5t$ $\quad g = 10$

$x(t) = -5t^2 + 5t$ $\quad -5t^2 + 5t = 0$ $\quad -5t(t-1) = 0$ $\quad t = 0, 1$ It takes 1 second.

2) $\int x''''(t) = \int 120$, $x(0)=0$, $x'(t) = 0$, $x''(t) = 0$, $x'''(t) = 0$, $x''''(t) = 0$ $\quad x(2) = ?$

$\int x^{(4)}(t) = \int 120t + C$ $\quad C = 0$ $\quad \int x^{(3)}(t) = \int 60t^2 + C$ $\quad C = 0$ $\quad \int x^{(2)}(t) = \int 20t^3 + C$ $\quad C = 0$

$\int x'(t) = \int 5t^4 + C$ $\quad x(t) = t^5 + C$ $\quad x(0) = 0 = 0 + C$ $\quad C = 0$

$\boxed{x(2) = 2^5 = 32 \text{ meters}}$

3) i) $x''(t) = -g$ $\quad x'(0) = 0$ $\quad x(0) = 100, g = 10$ $\quad x(t) = -\frac{gt^2}{2} + C$ $\quad x(0) = 100 = C$

$x(t) = 0 = -5t^2 + 100$ $\quad 20 = t^2$ $\quad t = \pm\sqrt{20}$ $\quad t = 2\sqrt{5}$

ii) Maple

4) a) $x(n) = f(x(n-1))$, $x(0) = x_0$

b) $[x_0, f(x_0), f(f(x_0)), \dots, f^{k-n}(x_0)]$ Plug in x_0, $f(x_0)$, $f(f(x_0))$ ... till $f^{k-n}(x_0)$

$[n, n+1, n+2, \dots\dots k]$ $\quad$ n to k, incrementing by 1.

c) The limit as $n \to \infty$ for $x(n) = \alpha$, α is the fixed point. $x = f(x)$ and solve for the solutions.

d) To find the stable fixed point use $x = f(x)$ to find the fixed points and plug those values into $|f'(x)|$. If the values are <1, its stable, $=1$ semistable & >1 unstable.

c) Starting nearby it will lead to the fixed point which is the local attractor.

5) a) Numerically plug in values close to the presumed stable fixed points into $f(x\pm 0.01)$ and over many iterations the value of $f(x_n + 0.01)$ will equal x_n
$f(x_0+0.01)$; $f(f(x_0+0.01))$ ...

b) Given $x(n) = f(x(n-1))$ plug in $x = f(x)$ → $x = f(x)$ and solve the discrete equilibrium solution of the newly formed equation. The zeros will be the fixed points of the equation.

c) First solve for the fixed points. Then take the derivative of the original discrete equation. $f'(x)$. Plug values into $f'(x)$ and if the absolute values are less than 1 it is stable. $|f'(x)|<1$ stable, $=1$ semistable, >1 unstable.

d) i a) Maple

b) $x = \frac{x+1}{x+2}$ $\quad x^2+2x = x+1$ $\quad x^2+x-1=0$ $\quad x = \frac{1\pm\sqrt{1+4}}{2}$

$x = \frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}$ ← Fixed

c) $f(x) = \frac{x+1}{x+2}$ $\quad \frac{x+2-(x+1)}{(x+2)^2}$ $\quad \frac{1}{x^2+4x+4}$ $\quad |f'(x_1)| \approx .14590$ $\quad |f'(x_2)| = 6.85$

x_1 is a stable fixed point

ii a) Maple

b) $x = \frac{5}{2}x(1-x)$ $\quad x = \frac{5}{2}x - \frac{5}{2}x^2$ $\quad \frac{5}{2}x^2 - \frac{3}{2}x = 0$ $\quad \frac{5}{2}x = \frac{3}{2}$ $\quad x(\frac{5}{2}x - \frac{3}{2}) = 0$

$x = 0,\ 0.6$ Fixed pts.

c) $f'(x) = \frac{5}{2}x(1-x) = \frac{5}{2} - 5x$ $\quad |f'(0)| = \frac{5}{2}$ $\quad |f'(0.6)| = \frac{1}{2}$

$x = 0.6$ is a stable fixed pt.

iii a) Maple

b) $x = \frac{7}{2}x - \frac{7}{2}x^2$ $\quad \frac{7}{2}x^2 - \frac{5}{2}x = 0$ $\quad x = 0, \frac{5}{7}$ Fixed pt.

c) $f'(x) = \frac{7}{2} - 7x$ $\quad |f'(0)| = \frac{7}{2}$ $\quad |f'(\frac{5}{7})| = \frac{3}{2}$

Neither $x = 0$ or $\frac{5}{7}$ are stable fixed points.

```
> #Hrudai Battini Hw24 Maple Portion
  read "/Users/hb334/Documents/DMB.txt";
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)

accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:

http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .

Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,

type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();

For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();

For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();

For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

```
> #3ii
  #x''(t) = -g+2*x'(t); x'(0)= 0, x(0) = 100;
  ode := diff(x(t),t,t) = -10+2*diff(x(t),t);
  dsolve({ode,x(0) = 100, D(x)(0)=0});
```

$$ode := \frac{d^2}{dt^2}\, x(t) = -10 + 2\, \frac{d}{dt}\, x(t)$$

$$x(t) = -\frac{5\, e^{2\,t}}{2} + 5\, t + \frac{205}{2} \qquad \textbf{(2)}$$

```
> #5d
  #i
```

```
evalf(Orb([(x+1)/(x+2)],[x],[0],100,105));
#ii
evalf(Orb([5/2*x*(1-x)],[x],[0.5],100,105));
#iii
evalf(Orb([7/2*x*(1-x)],[x],[0.6],100,105));
```

[[0.6180339887], [0.6180339887], [0.6180339887], [0.6180339887], [0.6180339887], [0.6180339887]]

[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000]]

[[0.5008842111], [0.8749972637], [0.3828196827], [0.8269407062], [0.5008842111], [0.8749972637]] **(3)**

```
>
```