Homework 24

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Preamble:

```
> read `C:/Users/cgrie/Dynam Models Bio/Homeworks/HW24/DMB.txt`
```

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This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

Problem 3:

Problem 3 (part 2)
A ball is dropped from a 100 m tall building. The air resistance is twice the mass times velocity. How long does it take for the ball to hit the ground.

Assume gravitation acceleration is 10 ms^-2

Initial conditions:
#Height
$IC1 := f(0) = 100$

$$D1 := f(0) = 100 \tag{2}$$

#Velocity
$IC2 := f'(0) = 0$

$$D2 := \mathrm{D}(f)(0) = 0 \tag{3}$$

#Acceleration
$IC3 := f''(0) = -10$

EQUATIONS:
#This is our acceleration, which
#Because Dr Z says that conversion of air resistance to the right unit can be achieved via a proportionality constant that has dimension s, my guess is that our proportionality constant is simply $\frac{1}{s}$, which is allowed to reside outside of the integrand and we can multiply it by our final result to make the units of time friendly?

$Eq1 := int(-10 + diff(f(t), t), t)$

$$Eq1 := -10\,t + f(t) \tag{4}$$

Problem 4

Define:
(a) A discrete-time dynamical system with 1 variable
$x_{n+1} = f(x_n)$, $n = 1, 2, \ldots$
(b) The Orbit of a discrete-time with one-quantity (variable) starting at
$x(n) = x_0$ up to $n = K$

(c) The notion of an equilibrium solution (Hint it is the solution of the difference equation that is always constant)

By definition, a fixed point exists at x, if and only if
$f(x) = x$
Examples of fixed points

$f(0) = 0$
$f([1, 1, 1]) = [1, 1, 1]$

Examples of mappings that are not fixed points

$F([1, 2, 3]) = [1, 2, 3]$ is not a fixed point because the components are unequal to each other:

$$\left(F_1([1, 2, 3]) = 1\right) \neq \left(F_2([1, 2, 3]) = 2\right) \neq \left(F_3([1, 2, 3]) = 3\right)$$

All systems of difference equations with multiple variables have their different components correspond to different days in a higher order difference equation. This means that
if today, yesterday, and the day before yesterday are not equal to each other, the condition of a fixed point $v_n = v_{n+1}$ for all n is broken.
$f(1) = 0$ is not a fixed point because yesterday and today are not equal

(d) The notion of a stable equilibrium solution

All recurrences with initial conditions in the neighborhood of a fixed point, will tend to one fixed point determine that such fixed point is stable
The notion of an unstable equilibrium solution:

The only orbit of cardinality 1 occurs when the initial condition is the fixed point. recurrences with Initial conditions that are not the fixed point will run away

Problem 5

(a) Describe how to numerically locate the stable fixed points using Orb

Finding the stable fixed points (if they exist) is easy Run the orb command and at a far away enough step, all subsequent values should equal each other after rounding.

To find an unstable fixed point, one would have to be wildly lucky with their initial conditions if algebra is not used or unable to be used.

(b) In terms of the underlying function, call it f(x), describe how to use algebra to find the set of fixed points.

(d) Apply

(i) $x(n) = \dfrac{x(n-1)+1}{x(n-1)+2}$

(a) Let underlying function be:

$$F := \frac{x[1]+1}{x[1]+2}$$

$$F := \frac{x_1 + 1}{x_1 + 2} \quad \textbf{(5)}$$

```
> Fsys:= ToSys(1,x,F);
```

$$Fsys := \left[\frac{x_1 + 1}{x_1 + 2}\right], [x_1] \quad \textbf{(6)}$$

```
>
  Orb(Fsys,[1.],1000,1010);
```

[[0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888]] **(7)**

It appears that 0.618033988 is a good approximation for the fixed point

Knowing that the system is ONE equation with degree ONE, whatever value pops up for a value of .

(b)As this is a nonlinear recurrence,

$$F(x) = x \equiv \frac{x+1}{x+2} = x \equiv x + 1 = x \times (x+2) \equiv x + 1 = x^2 + 2x \equiv 0 = x^2 + x - 1$$

And via the quadratic formula,

```
> solve(x^2+x-1=0);
```

$$\frac{\sqrt{5}}{2} - \frac{1}{2}, -\frac{1}{2} - \frac{\sqrt{5}}{2} \quad \textbf{(8)}$$

For confirmation that these are our fixed points

```
> #List of fixed points
  FP(Fsys);
```

$$\left\{\left[-\frac{1}{2} - \frac{\sqrt{5}}{2}\right], \left[\frac{\sqrt{5}}{2} - \frac{1}{2}\right]\right\} \quad \textbf{(9)}$$

Confirmed!

(ii) For recurrence

$$x(n) = \frac{5}{2} x(n-1)(1 - x(n-1))$$

We have underlying transformation

$$F2 := \frac{5}{2} x \cdot (1 - x)$$

$$F2 := \frac{5\,x\,(1-x)}{2} \tag{10}$$

$$F2 := \frac{5\,x\,(1-x)}{2} \tag{11}$$

By inspection, $x=0$ is a fixed point. Algebraically, x=0 is stable

```
> Orb([F2],[x],[0.],1,5);
```

$$[[0.], [0.], [0.], [0.], [0.]] \tag{12}$$

Algebraically, a fixed point is 0 and

(iii)For recurrence

3(i) A ball is dropped From A height of 100 Meters How Long Will it take To hit the ground?

Let initial Position
$x(0) = 100$

☆ Assuming Acceleration Due to Gravity is $-10\,ms^{-2}$, let $x''(t) = -10$

☆ Because the ball is dropped, LET Initial velocity $x'(t=0) = 0$

Step 1 $x'(t) = \int x''(t)\,dt = -10t + C_1$. Therefore, with I.C. $x'(0) = 0$ (Initial velocity), $0 = -10(0) + C_1$

Step 2 $x(t) = -10\int t\,dt = -\frac{10t^2}{2} + C_2$. Therefore, with IC. $x(0) = 100$ (Building height), $100 = -\frac{10(0)^2}{2} + C_2$ $\Rightarrow C_2 = 100$

Therefore, the Solution to $x(t) = -\frac{10t^2}{2} + 100$

When the ball hits the ground, $x(t_{ground}) = 0 \Rightarrow -5t^2 + 100 = 0$
$\Leftrightarrow -5(t^2 - 20) = 0 \Rightarrow$

Answer to 3(i)

It takes $\sqrt{20}$ Seconds for the dropped ball to hit the Ground