#not Okay to post

Anusha Nagar, 11.26.2021, Homework 24

① I didn't do anything wrong but ran out of time for algebraically solving (2.i) $x(t) = 0 = -5t^2 + 5t$ (would have gotten it with 20 more seconds)

② $a''' = 120 \ m/s^3$

$a = x''(t) \Rightarrow a'''(t) = x'''''(t)$

$$\begin{cases} x'''''(t) = 120, & x(0) = 0,\ x'(0) = 0,\ x''(0) = 0, \\ & x'''(0) = 0,\ x''''(0) = 0 \end{cases}$$

$x''''(t) = 120t + c$

$x''''(0) = 0 = c$

$x'''(t) = 60t^2 + c$

$x'''(0) = 0 = c$

$x''(t) = 20t^3 + c$

$x''(0) = 0 = c$

$x'(t) = 5t^4 + c$

$x'(0) = 0 = c$

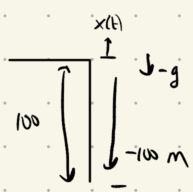
$x(t) = t^5 + c$

$x(0) = 0 = c$

$x(2) = 2^5 =$ 32 meters away from starting point after 2 seconds

③ $a(t) = -g = -9.81 = x''(t)$

$x'(t) = 0,\ x(0) = 0$



$x''(t) = -9.81 \Rightarrow x'(t) = -9.81t + c$

$x'(0) = c = 0$

$x(t) = \frac{-9.81}{2} t^2 + c$

$x(0) = 0 = c$

$x(t) = \frac{-9.81}{2} t^2$

$-100 = \frac{-9.81}{2} t^2$

$t = \sqrt{20.387} = 4.515$

(i) Ball reaches ground after 4.515 seconds

(ii) $\uparrow 2mv = 2mx'(t)$

$\downarrow -g$

$x''(t) = 2mx'(t) - g, \quad x'(0) = 0,\ x(0) = 0$

Find t when $x = -100$

From dsolve: (maple code attached to end of doc.)

$$x(t) = \frac{-981e^{2mt}}{400m^2} + \frac{981t}{200m} + \frac{981}{400m^2}$$

plug $x(t) = -100$ & solve for t

From maple:

$$t = -\frac{40000m^2 + 981\,\mathrm{LambertW}\left(-e^{\frac{-40000m^2}{981} - 1}\right) + 981}{1962m}$$

> #Problem 3 part ii

> $ODE := diff(diff(x(t), t), t) = 2 \cdot m \cdot diff(x(t), t) - 9.81$

$$ODE := \frac{d^2}{dt^2}\, x(t) = 2\, m \left(\frac{d}{dt}\, x(t)\right) - 9.81 \quad (1)$$

> $IC := x(0) = 0, D(x)(0) = 0$

$$IC := x(0) = 0, D(x)(0) = 0 \quad (2)$$

> $f := dsolve(\{ODE, IC\}, x(t))$

$$f := x(t) = -\frac{981\, e^{2\, m\, t}}{400\, m^2} + \frac{981\, t}{200\, m} + \frac{981}{400\, m^2} \quad (3)$$

> $help(subs)$

> $f2 := subs(x(t) = -100, f)$

$$f2 := -100 = -\frac{981\, e^{2\, m\, t}}{400\, m^2} + \frac{981\, t}{200\, m} + \frac{981}{400\, m^2} \quad (4)$$

> $solve(f2, t)$

$$-\frac{40000\, m^2 + 981\, \mathrm{LambertW}\left(-e^{-\frac{40000\, m^2}{981} - 1}\right) + 981}{1962\, m} \quad (5)$$

>

4 (a) Discrete-time dynamical system is one where we only need to know what happened yesterday (is a function of what happened yesterday) + is of form $x(n) = F(x(n-1))$, $x(0) = y$

(b) For an orbit of a discrete-time dynamical system, we plug in $F(x(n-1))$ into F again. For example, if we start @ $x(0)=x_0$, our orbit looks like $[x_0, F(x_0), F(F(x_0)), F(F(F(x_0))), \ldots]$
If we start @ $x(n) = x_0$ + go to $n=k$, we get $[x_0, F(x_0), F(F(x_0)), F(F(F(x_0))), F^4(x_0), \ldots F^K(x_0)]$

(c) I'm a little confused by what "notion of an equilibrium solution" means. But an equilibrium solution is one where if we start @ it, our orbit will always remain there. For discrete systems we find it by setting $F(x) = x$, while for continuous time we set $F(x) = 0$

(d) A stable equilibrium solution is one that we always return to within a neighborhood of the stable fixed point. We find it in discrete time, we see if $|F'(x)| < 1 \Rightarrow$ if so, it's stable. For continuous time, if $F'(x)$ is negative it's stable.

(5) (a) To numerically locate SFP, we compute the orbits for high numbers of n. If we use maple command, we could look @ n between 1000 + 1010 to see where our orbit ends up for various initial conditions. If the number is not changing $\Rightarrow$ stable fixed point

(b) To find set of fixed points, set $F(x) = x$ + solve for x. For example, if $F(x) = x(1-x)^2$, we set $x = x(1-x)^2 \Rightarrow x = 0, 2$

(c) For SFP, we compute $F'(x) \Rightarrow$ if absolute value $< 1 \Rightarrow$ stable. For each x (fixed point). For $F(x) = x(1-x)^2 \Rightarrow F'(x) = 1 - 4x + 3x^2$
$F'(0) = 1 \Rightarrow$ semi-stable
$F'(2) = 5 \Rightarrow$ unstable

(d) (i) $x(n) = \dfrac{x(n-1)+1}{x(n-1)+2}$

$F(x) = \dfrac{x+1}{x+2}$

a) SFP from maple: 0.6180339888

b) $x = \frac{x+1}{x+2}$

$x^2+2x-x-1=0$

$x^2+x-1=0$

$\frac{-1 \pm \sqrt{1-4(1)(-1)}}{2}$

$= -\frac{1}{2} \pm \frac{\sqrt{5}}{2} \Rightarrow FP = \{-\frac{1}{2}-\frac{\sqrt{5}}{2}, -\frac{1}{2}+\frac{\sqrt{5}}{2}\}$

(c) SFP:

$F'(x) = \frac{1}{x+2} - \frac{x+1}{(x+2)^2}$

In Maple, plug in $x = -\frac{1}{2}-\frac{\sqrt{5}}{2}, -\frac{1}{2}+\frac{\sqrt{5}}{2}$

$F'(-\frac{1}{2}-\frac{\sqrt{5}}{2}) = 6.854 \Rightarrow$ not $<1 \Rightarrow$ unstable

$F'(-\frac{1}{2}+\frac{\sqrt{5}}{2}) = 0.146 \Rightarrow <1 \Rightarrow$ stable!

$SFP = \{-\frac{1}{2}+\frac{\sqrt{5}}{2}\} = 0.6180339880$

matches result from Orb!

(ii) $x(n) = \frac{5}{2}x(n-1)(1-x(n-1))$

$F(x) = \frac{5}{2}x(1-x)$

a) From Orb:

SFP @ $x = 0.60$

b) $x = \frac{5}{2}x(1-x)$

$x = \frac{5}{2}x - \frac{5}{2}x^2$

$\frac{5}{2}x^2 - \frac{3}{2}x = 0$

$x(\frac{5}{2}x - \frac{3}{2}) = 0$

$x = 0, \frac{3}{5} \Rightarrow FP = \{0, \frac{3}{5}\}$

(c) $F'(x) = \frac{5}{2} - 5x$

$F'(0) = |\frac{5}{2}| \Rightarrow$ not $<1 \Rightarrow$ unstable

$F'(\frac{3}{5}) = \frac{5}{2} - 3 = |-\frac{1}{2}| < 1 \Rightarrow$ stable!

$SFP = \{\frac{3}{5}\} \Rightarrow$ matching Orb!

(iii) $x(n) = \frac{7}{2}x(n-1)(1-x(n-1))$

$F(x) = \frac{7}{2}x(1-x)$

a) From Orb: doesn't seem stable $\Rightarrow$ goes between

$0.8269 \rightarrow 0.501 \rightarrow 0.875 \rightarrow 0.383 \rightarrow 0.827 \rightarrow \ldots$

b) $x = \frac{7}{2}x(1-x)$

$x = \frac{7}{2}x - \frac{7}{2}x^2$

$\frac{7}{2}x^2 - \frac{5}{2}x = 0$

$x(\frac{7}{2}x - \frac{5}{2}) = 0$

$x = 0, \frac{5}{7}$

$FP = \{0, \frac{5}{7}\}$

(c) $F'(x) = \frac{7}{2} - 7x$

$F'(0) = |\frac{7}{2}|$ not $<1 \Rightarrow$ unstable

$F'(\frac{5}{7}) = |-\frac{3}{2}|$ not $<1 \Rightarrow$ unstable

$SFP = \{\ \}$ (empty set)

> #Problem 5 part D

> **read** "C://Users/an646/Documents/DMB.txt"

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(6)**

> #(i)

> $Help(Orb)$

Orb(F,x,x0,K1,K2): Inputs a transformation F in the list of variables x with initial point pt, outputs the trajectory of
of the discrete dynamical system (i.e. solutions of the difference equation): x(n)=F(x(n-1)) with x(0)=x0 from n=K1 to n=K2.
For the full trajectory (from n=0 to n=K2), use K1=0. Try:
*Orb([5/2*x*(1-x)],[x], [0.5], 1000,1010);*

(7)

Orb([(1+x+y)/(2+x+y),(6+x+y)/(2+4*x+5*y),[x,y], [2.,3.], 1000,1010); **(7)**

> $Orb\left(\left[\frac{(x+1)}{x+2}\right], [x], [5.], 1000, 1010\right)$

[[0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888], [0.6180339888]] **(8)**

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> $i\_f\_prime := \frac{1}{x+2} - \frac{(x+1)}{(x+2)^2}$

$$i\_f\_prime := \frac{1}{x+2} - \frac{x+1}{(x+2)^2} \quad \textbf{(9)}$$

> $evalf\left(subs\left(x = -\frac{1}{2} - \frac{\mathrm{sqrt}(5)}{2}, i\_f\_prime\right)\right)$

$$6.854101940 \quad \textbf{(10)}$$

> $evalf\left(subs\left(x = -\frac{1}{2} + \frac{\mathrm{sqrt}(5)}{2}, i\_f\_prime\right)\right)$

$$0.1458980339 \quad \textbf{(11)}$$

> $evalf\left(-\frac{1}{2} + \frac{\mathrm{sqrt}(5)}{2}\right)$

$$0.6180339880 \quad \textbf{(12)}$$

>

>

> #ii

> $Orb\left(\left[\frac{5}{2} \cdot x \cdot (1-x)\right], [x], [0.5], 1000, 1010\right)$

[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000]] **(13)**

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> #iii

> $Orb\left(\left[\frac{7}{2} \cdot x \cdot (1-x)\right], [x], [0.75], 1000, 1010\right)$

[[0.8269407062], [0.5008842111], [0.8749972637], [0.3828196827], [0.8269407062], [0.5008842111], [0.8749972637], [0.3828196827], [0.8269407062], [0.5008842111], [0.8749972637]] **(14)**

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