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Anne Somalwar, hw24, 11/26/2021

2) $a'''(t) = 120$, $v(0) = 0$, $a(0) = 0$, $a'(0) = 0$
$a''(0) = 0$

$a''(t) = 120t$

$a'(t) = 60t^2$

$a(t) = 20t^3$

$v(t) = 5t^4$

$x(t) = t^5 + x_0$

$x(2) = 2^5 + x_0$

The particle is 32 meters away from the starting pt.

3) (i) $a(t) = -10$ $\quad v(0) = 0, \quad x(0) = 100$

$v(t) = -10t + v_0 = -10t$

$x(t) = -5t^2 + x_0 = -5t^2 + 100$

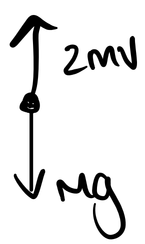
$-5t^2 + 100 = 0$

$5t^2 = 100$

$t^2 = 20$

$\boxed{t = \sqrt{20} \text{ seconds}}$

(ii)



$F = ma = -mg + 2mv$ $\quad v(0) = 0, \quad x(0) = 100$

$v'(t) = -10 + 2v$

$\boxed{x''(t) = -10 + 2x'(t)}$

SEE MAPLE CODE

4) (a) $x(n) = f(x(n-1), x(n-2), \ldots, x(0))$

(b) The orbit consists of all the values of $x(i)$ from $i=n$ to $i=k$ starting at $x(n) = x_0$.

(c) $x(n) = x(n-1)$

(d) a fixed pt x_0 such that there exists a neighborhood $(x_0-\delta, x_0+\delta)$ such that if you start at $x(0) \in (x_0-\delta, x_0+\delta)$,

Then $\lim_{n\to\infty} x(n) = x_0$.

5)

(a) If the result returned by Orb where k1 and k2 are large numbers is virtually unchanging, then that result is a stable fixed point. Run Orb many times, incrementing the initial point by small values to find them all.

(b) Set $x = f(x)$ and solve for x.

(c) If x_0 is a fixed pt. and $|f'(x_0)| < 1$, then x_0 is stable.

(d)

(i) (a) SEE MAPLE CODE

(b) $x = \frac{x+1}{x+2}$

$x^2+2x = x+1$

$x^2+x-1=0$

$\frac{-1 \pm \sqrt{1+4}}{2} = \boxed{\frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}}$

(c) $f(x) = \frac{x+1}{x+2}$

$f'(x) = \frac{(x+2)-(x+1)}{(x+2)^2} = \frac{1}{(x+2)^2}$

$f'\left(\frac{-1+\sqrt{5}}{2}\right) = \frac{1}{\left(\frac{-1+\sqrt{5}}{2}+2\right)^2} < 1$, so

$\boxed{\frac{-1+\sqrt{5}}{2} \text{ is stable}}$

$$f'\left(\frac{-1-\sqrt{5}}{2}\right) = \frac{1}{\left(\frac{-1-\sqrt{5}}{2}+2\right)^2} > 1 \text{ , so}$$

$$\boxed{\frac{-1+\sqrt{5}}{2} \text{ is not stable}}$$

(ii) (a) SEE MAPLE CODE

(b) $x = \frac{5}{2}x(1-x)$

$x=0$

$1 = \frac{5}{2}(1-x)$

$\frac{2}{5} = 1-x$

$x = 3/5$

$$\boxed{\text{fixed pts: } x = 0, 3/5}$$

(c) $f(x) = \frac{5}{2}x - \frac{5}{2}x^2$

$f'(x) = \frac{5}{2} - 5x$

$f'(0) = \frac{5}{2} > 1$, so $\boxed{0 \text{ is not stable.}}$

$f'(3/5) = \frac{5}{2} - 3 = \frac{5}{2} - \frac{6}{2} = -\frac{1}{2}$

$|-\frac{1}{2}| < 1$, so $\boxed{3/5 \text{ is stable}}$

(iii) (a) SEE MAPLE CODE

(b) $x = \frac{7}{2}x(1-x)$

$x = 0$

$1 = \frac{7}{2}(1-x)$

$\frac{2}{7} = 1 - x$

$x = \frac{5}{7}$

fixed pts: $\boxed{x = 0, \frac{5}{7}}$

(c) $f(x) = \frac{7}{2}x - \frac{7}{2}x^2$

$f'(x) = \frac{7}{2} - 7x$

$f'(0) = \frac{7}{2} > 1$, so $\boxed{0 \text{ is not stable}}$

$f'(\frac{5}{7}) = \frac{7}{2} - 5 = \frac{7}{2} - \frac{10}{2} = -\frac{3}{2}$

$|-\frac{3}{2}| > 1$, so $\boxed{\frac{5}{7} \text{ is not stable}}$