HW 24

1) I got the second question wrong on the attendance quiz b/c I failed to properly translate the physics problem into differential equations.

2) $a'''(t) = 120 \text{ m/s}^3$ | $v(0) = 0$

$a''(t) = 120t$ | $a(0) = v'(0) = 0$

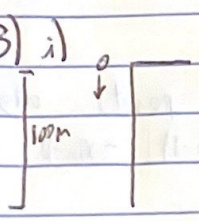
$a'(t) = 60t^2$ | $a'(0) = 0$

$a(t) = 20t^3$ | $a''(0) = 0$

$v(t) = 5t^4$

$x(t) = t^5$ | $\boxed{x(2) = 32 \text{ m}}$

3) i)



$a(t) = -10$

$v(t) = -10t$

$x(t) = -5t^2$

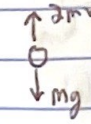
$100 = 5t^2$

$t = \sqrt{20} = \boxed{4.47 \text{ s}}$

$x = x_0 + v_0 + \frac{a}{2}t^2$

$100 = (-5)t^2$

ii)



$mg - 2mv = ma$ $10 - 2x'(t)$

$x''(t) = 10 - 2x'(t)$ | $a = g - 2v = 10 - 2v$

$x(0) = 0$ | ~~$x(t) = 10t - 2vt$~~

$v(0) = 0$ | ~~$x(t) = 5t^2 - vt^2$~~

$\boxed{= 20.5 \text{ s}}$

▲See Maple PDF★

4) a) A differa equation of only 1st order w/ 1 quantity

format → $x(n) = f(x(n-1))$

b) The orbit is all the terms of the differa equation when x_0 is plugged in, all the way to x_K

c) An ~~M~~ equilibrium solution is a point at which if you start then, you will always end up there no matter the # of terms. $x = f(x(n-1))$

d) A stable fixed point is when there is a basin of attraction around the equilibrium point when if you start

anywhere in that neighborhood you will end up at that stable fixed point. $f'(x) < |1|$

5) a) Using Orb, you can numerically figure out what the function's long term behavior is going. The long term behavior of the system will be its fixed points, and if it stays constant then it is stable.

b) Set underlying function equal to x and solve.

$f(x) = x$

c) Take the derivative of the function, plug in equilibria and iff $f'(x) < |1|$ then that point is stable

d) i) a) Orb($[\frac{x+1}{x+2}]$, [x], [0.5], 100, 1010)

$= 0.618$

b) $x = \frac{x+1}{x+2}$

Solve(↓ , x) = $0.618, -1.618$

c) $f'(x) = \frac{1}{x+2} - \frac{x+1}{(x+2)^2}$

$f'(0.618) = 0.146 < 1$ ∴ stable

ii) a) Orb($[\frac{5}{2} \cdot x \cdot (1-x)]$, [x], [0.5], 1000, 1010)

$= 0.600$

b) $x = \frac{5}{2} x(1-x)$

Solve(↓ , x) = $0, 0.600$

c) $f'(x) = \frac{5}{2} - 5x$

$f'(0) = \frac{5}{2} > 1$ ⇒ not stable

$f'(0.6) = -0.5 < 1$ ⇒ stable

iii) a) Orb($[\frac{7}{2} \cdot x \cdot (1-x)]$, [x], [0.5], 100, 1010)

$= 0.500, 0.875, 0.383$

b) $x = \frac{7}{2} x(1-x)$

$x = 0, 0.714$

c) $f'(x) = \frac{7}{2} - 7x$

$f'(0) = \frac{7}{2} > 1$ ⇒ not stable

$f'(0.71) = -1.99 < 1$ ⇒ stable