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> #OK to post Homework
  #Shreya Ghosh, 11-22-2021, Assignment 22
> read "/Users/shreyaghosh/Documents/DMB.txt"
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First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

```
> #2a.
> #System: x(n)=2x(n-1)+3y(n-1), y(n)=3x(n-1)+y(n-1)
> Orb([2·x + 3·y, 3·x + y], [x, y], [20, 10], 0, 9)
```

[[20, 10], [70, 70], [350, 280], [1540, 1330], [7070, 5950], [31990, 27160], [145460, 123130], [660310, 559510], [2999150, 2540440], [13619620, 11537890], [61852910, 52396750]] **(2)**

```
> #There were 13619620 lynxes and 11537890 hares at the start of year 10
>
> #2b.
```

> $\#System: x'(t)=2x+3y, y'(t)=3x+y$

> $sys := diff(x(t), t) = 2\cdot x(t) + 3\cdot y(t), diff(y(t), t) = 3\cdot x(t) + y(t)$

$$sys := \frac{d}{dt} x(t) = 2\,x(t) + 3\,y(t), \frac{d}{dt} y(t) = 3\,x(t) + y(t) \quad (3)$$

> $f := \{x(t), y(t)\}$

$$f := \{x(t), y(t)\} \quad (4)$$

> $evalf(dsolve(\{sys, x(0) = 20, y(0) = 10\}, f))$

$$\{x(t) = 16.57595949\,e^{4.541381265\,t} + 3.424040509\,e^{-1.541381265\,t}, y(t) = 14.04194430\,e^{4.541381265\,t} - 4.041944304\,e^{-1.541381265\,t}\} \quad (5)$$

> $evalf(subs(t = 10, 16.57595949\cdot \exp(4.541381265\cdot t) + 3.424040509\cdot \exp(-1.541381265\cdot t)))$

$$8.758846449 \times 10^{20} \quad (6)$$

> $evalf(subs(t = 10, 14.04194430\cdot \exp(4.541381265\cdot t) - 4.041944304\cdot \exp(-1.541381265\cdot t)))$

$$7.419856090 \times 10^{20} \quad (7)$$

> #There were $8.758846449\times 10^{20}$ *lynxes* **and** $7.419856090 \times 10^{20}$ *hares* **in** *year* 10

>

> #3.

> $F := AllenSIR(1, .3, .2, x, y)$

$$F := [0.5\,x + y\,(1 - e^{-x}), 0.3 - 0.3\,y + y\,e^{-x}] \quad (8)$$

> $OrbF(F, [x, y], [.3, .5], 1000, 1010)$

[[0.2594083191, 0.5676528016], [0.2594083191, 0.5676528016], [0.2594083191, 0.5676528016], [0.2594083191, 0.5676528016], [0.2594083191, 0.5676528016], [0.2594083191, 0.5676528016], [0.2594083191, 0.5676528016], [0.2594083191, 0.5676528016], [0.2594083191, 0.5676528016], [0.2594083191, 0.5676528016], [0.2594083191, 0.5676528016]] (9)

> $evalf\left(subs\left(x = 0.2594083191, y = 0.5676528016, x = \frac{y\cdot(1 - \exp(-x))}{0.5}\right)\right)$

$$0.2594083191 = 0.2594083190 \quad (10)$$

> $evalf\left(subs\left(x = 0.2594083191, y = 0.5676528016, y = 1 - x\cdot\left(1 + \frac{0.2}{0.3}\right)\right)\right)$

$$0.5676528016 = 0.5676528014 \quad (11)$$

>

> $F := AllenSIR(.8, .4, .3, x, y)$

$$F := [0.3\,x + y\,(1 - e^{-0.8\,x}), 0.4 - 0.4\,y + y\,e^{-0.8\,x}] \quad (12)$$

> $OrbF(F, [x, y], [.6, .1], 1000, 1010)$

[[0.0594452853, 0.8959707507], [0.0594452853, 0.8959707507], [0.0594452853, 0.8959707507], [0.0594452853, 0.8959707507], [0.0594452853, 0.8959707507], [0.0594452853, 0.8959707507], [0.0594452853, 0.8959707507], [0.0594452853, 0.8959707507], [0.0594452853, 0.8959707507], [0.0594452853, 0.8959707507], [0.0594452853, 0.8959707507]] (13)

> $evalf\left(subs\left(x = 0.0594452853, y = 0.8959707507, x = \frac{y\cdot(1-\exp(-0.8\cdot x))}{0.7}\right)\right)$

$$0.0594452853 = 0.059445285 \quad (14)$$

> $evalf\left(subs\left(x = 0.0594452853, y = 0.8959707507, y = 1 - x\cdot\left(1+\frac{0.3}{0.4}\right)\right)\right)$

$$0.8959707507 = 0.8959707507 \quad (15)$$

>

> #4.

> $F := AllenSIR(1.2, 1, 0, x, y)$

$$F := \left[y\left(1-e^{-1.2\,x}\right), 1 - y + y\,e^{-1.2\,x}\right] \quad (16)$$

> $OrbF(F, [x, y], [.8, .8], 1000, 1010)$

[[0.1103002718, 0.8896997282], [0.1103002718, 0.8896997282], [0.1103002718, 0.8896997282], [0.1103002718, 0.8896997282], [0.1103002718, 0.8896997282], [0.1103002718, 0.8896997282], [0.1103002718, 0.8896997282], [0.1103002718, 0.8896997282], [0.1103002718, 0.8896997282], [0.1103002718, 0.8896997282], [0.1103002718, 0.8896997282]] (17)

> $evalf\left(subs\left(x = 0.1103002718, y = 0.8896997282, x = \frac{(1-x)\cdot(1-\exp(-1.2\cdot x))}{1}\right)\right)$

$$0.1103002718 = 0.1103002718 \quad (18)$$

>

> $F := AllenSIR(.8, .4, 0, x, y)$

$$F := \left[0.6\,x + y\left(1-e^{-0.8\,x}\right), 0.4 - 0.4\,y + y\,e^{-0.8\,x}\right] \quad (19)$$

> $OrbF(F, [x, y], [.8, .8], 1000, 1010)$

[[0.4128852218, 0.5871147782], [0.4128852218, 0.5871147782], [0.4128852218, 0.5871147782], [0.4128852218, 0.5871147782], [0.4128852218, 0.5871147782], [0.4128852218, 0.5871147782], [0.4128852218, 0.5871147782], [0.4128852218, 0.5871147782], [0.4128852218, 0.5871147782], [0.4128852218, 0.5871147782], [0.4128852218, 0.5871147782]] (20)

> $evalf\left(subs\left(x = 0.4128852218, y = 0.5871147782, x = \frac{(1-x)\cdot(1-\exp(-0.8\cdot x))}{0.4}\right)\right)$

$$0.4128852218 = 0.412885222 \quad (21)$$

>

HW 22

1. I solved the matrix problem the long way.

i. a) $\begin{bmatrix}1\\1\end{bmatrix}$ b) $\begin{bmatrix}2\\1\end{bmatrix}$ c) $\begin{bmatrix}-1\\1\end{bmatrix}$ d) $\begin{bmatrix}3\\2\end{bmatrix}$

$\begin{bmatrix}2&1\\1&2\end{bmatrix}$

$Av_a = \begin{bmatrix}2&1\\1&2\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\3\end{bmatrix} = 3\begin{bmatrix}1\\1\end{bmatrix}$

v_a is an eigenvector with eigenvalue 3

$Av_b = \begin{bmatrix}2&1\\1&2\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}5\\4\end{bmatrix}$

v_b is not an eigenvector

$Av_c = \begin{bmatrix}2&1\\1&2\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}-1\\1\end{bmatrix}$

v_c is an eigenvector with eigenvalue 1

$Av_d = \begin{bmatrix}2&1\\1&2\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix} = \begin{bmatrix}8\\7\end{bmatrix}$

v_d is not an eigenvector

ii. a) $\begin{bmatrix}1\\1\end{bmatrix}$ b) $\begin{bmatrix}\frac{1}{2}\\1\end{bmatrix}$ c) $\begin{bmatrix}2\\1\end{bmatrix}$ d) $\begin{bmatrix}3\\2\end{bmatrix}$

$\begin{bmatrix}-3&2\\-2&2\end{bmatrix}$

$Av_a = \begin{bmatrix}-3&2\\-2&2\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}-1\\0\end{bmatrix}$

v_a is not an eigenvector

$Av_b = \begin{bmatrix}-3&2\\-2&2\end{bmatrix}\begin{bmatrix}\frac{1}{2}\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{2}\\1\end{bmatrix}$

v_b is an eigenvector with eigenvalue 1

$Av_c = \begin{bmatrix}-3&2\\-2&2\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}-4\\-2\end{bmatrix} = -2\begin{bmatrix}2\\1\end{bmatrix}$

v_c is an eigenvector with eigenvalue -2

$Av_d = \begin{bmatrix}-3&2\\-2&2\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix} = \begin{bmatrix}-5\\-2\end{bmatrix}$

v_d is not an eigenvector

5. $x(n) = \frac{x(n-1)}{10+x(n-1)}$

$F = \frac{x}{10+x}$

$x = \frac{x}{10+x}$

$10x + x^2 = x$

$x^2 + 9x = 0$

$x(x+9) = 0 \Rightarrow x = 0, -9$

$F' = \frac{1(x+10) - x(1)}{(x+10)^2}$

$= \frac{10}{(x+10)^2}$

$F'(0) = \frac{10}{100} = \frac{1}{10} \Rightarrow$ stable b/c $\frac{1}{10} < 1$

$F'(-9) = \frac{10}{1} = 10 \Rightarrow$ not stable b/c $10 > 1$