OK to post

Anne Somalwar, hw22, 11/22/2021

6) <u>Proof</u>: First we will show that,

if $x(0) = x_0 \neq -10$, then,

$$x(n) = \frac{x_0}{10^n + \left(\frac{1-10^n}{1-10}\right)x_0}.$$

With this equation for $x(n)$, we

have $$x(n-1) = \frac{x_0}{10^{n-1} + \left(\frac{1-10^{n-1}}{1-10}\right)x_0}.$$

Then, $$\frac{x(n-1)}{10 + x(n-1)} = \frac{\frac{x_0}{10^{n-1} + \left(\frac{1-10^{n-1}}{1-10}\right)x_0}}{10 + \frac{x_0}{10^{n-1} + \left(\frac{1-10^{n-1}}{1-10}\right)x_0}}$$

$$= \frac{x_0}{10\left(10^{n-1} + \left(\frac{1-10^{n-1}}{1-10}\right)x_0\right) + x_0}$$

$$= \frac{x_0}{10^n + \frac{10-10^n}{1-10} x_0 + x_0}$$

$$= \frac{x_0}{10^n + \frac{1-10^n}{1-10} x_0} = x(n).$$

Thus, $x(n) = \dfrac{x_0}{10^n + \left(\frac{1-10^n}{1-10}\right) x_0}$.

Then, $x(n) = \dfrac{x_0}{10^n + \frac{10^n - 1}{9} x_0} = \dfrac{x_0}{10^n + \frac{10^n x_0}{9} - \frac{x_0}{9}}$

$$= \frac{x_0}{10^n \left(1 + \frac{x_0}{9} - \frac{x_0}{10^n \cdot 9}\right)}.$$

Thus, $\lim_{n\to\infty} x(n) = \lim_{n\to\infty} \dfrac{x_0}{10^n\left(1+\frac{x_0}{9}-\frac{x_0}{10^n\cdot 9}\right)}$

$= 0$. Thus, 0 is a global attractor. //