Erratum to the paper "Logarithmic intertwining operators and associative algebras"

Yi-Zhi Huang and Jinwei Yang

In the paper [HY], the statement of Lemma 5.5 is wrong since in general $\rho(\mathcal{Y})((L(-1)+L(0)_s)w_{(1)}\otimes w_{(2)})\neq 0$ for $w_{(1)}\in W_1$.

For Lemma 5.5 to hold, we need to modify the following definitions: The definition of $w*_N v$ on Page 1473 should be

$$w *_N v = \sum_{m=0}^{N}(-1)^m\binom{m+N}{N}\mathrm{Res}_x x^{-N-m-1}\cdot \\ \cdot (1+x)^{-(L_W(-1)+L_W(0))}Y_{WV}^W((1+x)^{L_W(0)+N}w, x)v.$$

for $N\in\mathbb{N}$, $v\in V$ and $w\in W$. The subspace $O_N(W)$ of W on the same page should be defined as the subspace spanned by elements of the form

$$\mathrm{Res}_x x^{-2N-2}Y_W((1+x)^{L_V(0)+N}v, x)w$$

for $v\in V$ and $w\in W$, where ω is the conformal element of V. Note that in these definitions, there are two modifications: (i) In the definition of $w*_N v$, $Y_{WV}^W((1+x)^{L_W(0)_s+N}w, x)v$ should be $(1+x)^{-(L_W(-1)+L_W(0))}Y_{WV}^W((1+x)^{L_W(0)+N}w, x)v$. (ii) $O_N(W)$ should not contain $(L_W(-1)+L_W(0)_s)w$.

With the corresponding formulas involving things as in the items (i) and (ii) being modified, all the results still hold and only the proofs of Lemma 4.4, Lemma 4.6, Theorem 4.7 need minor modifications. All the constructions, results and proofs, including Lemma 5.5, are correct now without any modifications needed.

In fact, besides using those results proved in Section 4, the minor modifications needed in the proofs of Lemma 4.4, Lemma 4.6, Theorem 4.7 are adding the operator $(1+x)^{-(L_W(-1)+L_W(0))}$ and changing all $L(0)_s$ to $L(0)$ when the right action $w*_N v$ is involved and in the proofs, using the formulas

$$(1+x)^{-(yL_W(-1)+L_W(0))} = (1+x)^{-L(0)}e^{-xyL(-1)}$$
$$(1+x)^{L(0)}\mathcal{Y}(w, y)(1+x)^{-L(0)} = \mathcal{Y}((1+x)^{yL(-1)+L(0)}w, y)$$

for an intertwining operator $\mathcal{Y}$, including in particular the vertex operators Y_W and Y_{WV}^W and other known familiar formulas. Because of (ii), some parts of these proofs should be deleted completely. The details have been given in the new arXiv version arXiv:1104.4679v3 of the paper [HY].

Without the modifications above, the results after Lemma 5.5 in [HY] and thus all the results in [HY] hold in the case that the eigenvalues of $L(0)$ on W_2 and W_3 are all congruent to a common complex number modulo $\mathbb{Z}$ and the nilpotent parts of $L(0)$ on W_2 and W_3 are 0.

For a general theory on associative algebras and intertwining operators, see [H]. The corrected definitions above can be obtained easily from a subspace of the $A^\infty(V)$-bimodule $A^\infty(W)$ in [H].

References

[H] Y.-Z. Huang, Associative algebras and intertwining operators, to appear; arXiv:2111.06943.

[HY] Y.-Z. Huang and J. Yang, Logarithmic intertwining operators and associative algebras, *J. Pure Appl. Alg.* **216** (2011), 1467–1492.

Department of Mathematics, Rutgers University, 110 Frelinghuysen Rd., Piscataway, NJ 08854-8019

E-mail address: yzhuang@math.rutgers.edu

School of Mathematical Sciences, Shanghai Jiao Tong University, 800 Dongchuan Rd., Shanghai 200240, China

E-mail address: jinwei2@sjtu.edu.cn