640: 560 Homological Algebra Fall 2003

The Theory and Apllications of Tensor Categories

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The theory of tensor categories has recently become an important tool in the study of a number of mathematical and physical problems. The fundamental connections among quantum groups, knot and three-dimensional invariants and conformal field theories are best understood through "modular" tensor categories. Many classical algebraic notions, for examples, associative algebras and Frobenius algebras, have generalizations in the framework of tensor categories. More recently, tensor categories and these generalizations of classical algebraic notions have also been used to study boundary conformal field theories and D-branes in string theory.

This course is an introduction to the theory of tensor categories and its applications in representation theories, quantum groups, knot invariants and conformal field theories. I will start with a review of basic notions in the theory of categories. The theories and applications of monoidal categories, tensor categories, symmetric tensor categories, braided tensor categories and modular tensor categories will be discussed next. Then I plan to discuss examples of modular tensor categories constructed from representations of quantum groups and vertex operator algebras.

Prerequisites: I will assume that the students have some basic knowledge in algebra, as covered in the first-year graduate courses.

Time: Monday and Wednesday, 1:10 – 2:30 pm.

Room: Hill 423.

Text: (i) *Categories for the Working Mathematician* by Saunders Mac Lane (Graduate Texts in Mathematics, Vol. 5, Springer-Verlag, New York, 1971);

(ii) *Lectures on Tensor Categories and Modular Functors* by Bojko Bakalov and Alexander Kirillov, Jr. (University Lecture Series, Vol. 21, American Mathematical Society, Providence, 2001);

(iii) Some papers to be distributed.