640:555 Topics in Algebra Spring 2006

Fusion rules, modular transformations and the Verlinde formula

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One of the most important discoveries by physicists in two-dimensional conformal field theory is the famous relation between the fusion rules and the action of the modular transformation $\tau \mapsto -1/\tau$ on the space of vacuum characters. It states that this action of the modular transformation diagonalizes the matrices formed by the fusion rules. This relation was first conjectured by E. Verlinde. Combined with axioms for higher-genus rational conformal field theories, the Verlinde conjecture lead to a Verlinde formula for the dimensions of the spaces of conformal blocks on higher-genus Riemann surfaces. In the particular case of the conformal field theories associated to affine Lie algebras (the Wess-Zumino-Novikov-Witten models), this Verlinde formula gives a surprising formula for the dimensions of the spaces of sections of the "generalized theta divisors" and has led to many exciting developments and deep mathematics.

Assuming the axioms for rational conformal field theories, Moore and Seiberg proved this Verlinde conjecture by deriving a fundamental set of polynomial equations. However, since there is no construction of rational conformal field theories satisfying the axioms needed in the arguments of Moore and Seiberg, precise formulations and mathematical proofs of the Verlinde conjecture and the Verlinde formula is still needed. In the particular case of the Wess-Zumino-Novikov-Witten models, the Verlinde formula was studied by many people and was proved by Beauville-Laszlo and Faltings using the work of Tsuchiya-Ueno-Yamada and Kumar-Narasimhan-Ramanathan.

Recently a precise formulation and a proof of the Verlinde conjecture in the general case has been obtained. In this course, I will discuss the formulation, the proof and the application of this general version of the Verlinde conjecture.

Prerequisites: I will assume that the students have some basic knowledge in algebra and complex variables, as covered in the first-year graduate courses, and some basic knowledge in vertex operator algebra theory, as covered in the course given by Lepowsky in Fall, 2005.

Time: Monday and Wednesday, 3:20 – 4:40 pm.

Room: Hill 525.

Text: There is no textbook available. Some expository and research papers will be distributed.