Formula sheet for the final exam, Math 421

Laplace Transform

Function	**Laplace transform**
$f(t)$	$\mathcal{L}(f(t)) = F(s) = \int_0^\infty e^{-st} f(t)dt$
$af(t) + bg(t)$	$aF(s) + bG(s)$
t^n for a nonnegative integer n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin(kt)$	$\frac{k}{s^2+k^2}$
$\cos(kt)$	$\frac{s}{s^2+k^2}$
$e^{at}f(t)$	$F(s-a)$
$f(t-a)\mathcal{U}(t-a)$	$e^{-as}F(s)$
$g(t)\mathcal{U}(t-a)$	$e^{-as}\mathcal{L}(g(t+a))$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
$\int_0^t f(w)dw$	$\frac{F(s)}{s}$
$(f * g)(t) = \int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$
$\delta(t-a)$	e^{-as}
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}F(s)$
$f(t+T) = f(t)$ (periodic)	$\frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)dt$

Trigonometric Formulas

$$\begin{aligned}\sin(A+B) &= \sin A\cos B + \cos A\sin B\\ \cos(A+B) &= \cos A\cos B - \sin A\sin B\end{aligned}$$

Fourier Series

For $f(x)$ defined on $[-L, L]$, the Fourier series of $f(x)$ is

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right).$$

Fourier Coefficients

$$\begin{aligned}a_n &= \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx,\\ b_n &= \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx.\end{aligned}$$

Orthogonality

$$\begin{aligned}\int_{-L}^{L}\sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right)dx &= \begin{cases}0 & m\neq n,\\ L & m=n,\end{cases}\\ \int_{-L}^{L}\cos\left(\frac{m\pi x}{L}\right)\cos\left(\frac{n\pi x}{L}\right)dx &= \begin{cases}0 & m\neq n,\\ L & m=n\neq 0,\\ 2L & m=n=0,\end{cases}\\ \int_{-L}^{L}\cos\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right)dx &= 0.\end{aligned}$$