Math 152, Fall, 2004, Workshop 9

Honors Section

1. For each of the following series, determine whether the three conditions of the alternating series test are satisfied. If the answer is 'YES', determine how many terms of the series need be added up to get an approximation to the exact sum of the series within an error tolerance of 10^{-20}.

$$(i)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{(n^3+4)^2}, \qquad (ii)\ \sum_{n=2}^{\infty} \frac{(-1)^n n}{n+\ln n}, \qquad (iii)\ \sum_{n=1}^{\infty} (-1)^n \frac{\sin^2 n}{n}.$$

2. Consider the two series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$

and

$$\sum_{n=1}^{\infty} \frac{1}{n2^n}.$$

   (a) Explain why each series converges.

   (b) It happens that the sum of each of these series equals $\ln 2$. (We shall learn why, in another week or so.) Granted this fact, suppose that you have to compute $\ln 2$ to an accuracy of 10^{-6}. Which series is better suited for this calculation? Justify your answer by figuring out how many terms of each series you have to add up to get a partial sum of the desried accuracy, that is, to have $|R_N| < 10^{-6}$. (Hint for the second series: Estimate the error $|R_N$ by comparison with the error of a certain geometric series.)

3. Consider the alternating series

$$\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{n} + \frac{(-1)^n}{n}\right).$$

   (a) Write down explicitly the first 10 terms of the series.

   (b) Verify that $a_n \geq 0$ and $\lim_{n\to\infty} a_n = 0$.

(c) Does the series converge? Explain.

(d) Is the alternating series test applicable? Explain.

4. For which positive integers k does

$$\sum_{n=1}^{\infty} \frac{k^n n!}{n^n}$$

converge? Hint:

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right) = e.$$