# Math 152, Fall, 2004, Workshop 7

## Honors Section

1. Consider the sequences

$$a_n = \left(1+\frac{1}{n}\right)^n,\ b_n = \left(1+\frac{1}{n^2}\right)^n,\ a_n = \left(1+\frac{1}{n}\right)^{\sqrt{n}}.$$

   (a) Using your calculator (try the TABLE feature; TblSet is useful, too), find and plot the first ten terms of each sequence. Using this information, try to guess the limit of each sequence.

   (b) Use L'Hôpital's Rule to find the limit of each sequences. (Hint: take logarithms, then replace n by $x$ and let $x$ tend to infinity, or replace $n$ by $1/x$ and let $x$ tend to zero.)

2. Suppose that you draw a $2'' \times 2''$ square, then join the midpoints of its sides to make another square, then join the midpoints of that square's sides to form an- other square, etc., and that in fact you draw an infinite sequence of squares in this way. What is the sum of the perimeters of all the squares?

3. Two of the following statements are (always) true, and two are (sometimes) false. Decide which are which. Then for each true statement, explain briefly why it must be true; for each false statement, give an example of a series for which it is not true.

   (a) If a series diverges, then its $n$-th term does not approach 0 as $n \to \infty$.

(b) If the $n$-th term of a series does not approach 0 as $n \to \infty$, then the series diverges.

(c) If a series converges, then its partial sums form a bounded sequence.

(d) If the partial sums of a series form a bounded sequence, then the series converges.