Math 152, Fall, 2004, Workshop 5

Honors Section

1. (a) Use the Trapezoidal Rule with $n = 4$ to approximate

$$\int_0^1 -\cos(x^2 + 3)dx.$$

Your answer should contain 4 digits past the decimal point (e.g. 1.2345).

(b) How good is the approximation obtained above?

(c) How large must n be so that the approximation given by the Trapezoidal Rule is correct to 4 digits past the decimal point (i.e. so that $|E_{T_n}| < 0.0001$)?

(**Some useful inequalities:**

(a) **Triangle inequality** $|f(x) + g(x)| \leq |f(x)| + |g(x)|$; $|f(x) - g(x)| \leq |f(x)| + |g(x)|$.

(b) If $|f(x)| \leq F$ and $H \leq |g(x)| \leq G$, then $|f(x)g(x)| \leq FG$ and $\dfrac{|f(x)|}{|g(x)|} \leq \dfrac{F}{H}$.

(c) If $f(x)$ is increasing on $[a, b]$, then $f(x) \leq f(b)$; if $f(x)$ is decreasing on $[a, b]$, then $f(x) \leq f(a)$.

(d) $|\sin x| \leq 1$ and $|\cos x| \leq 1$ for all real numbers.

These principles can be used to find an upper bound for a function quickly, especially when trying use the Differential Calculus to maximize that function is likely to be quite arduous.)

2. Assume that a is a positive constant. Let R be the region bounded above by $y = 1/x^a$, below by $y = 0$, and on the left by the line $x = 1$. Determine those values of a for which the integral that results from attempting to calculate each of the following converges:

(a) The volume of the solid obtained by rotating R around the x-axis.

(b) The volume of the solid obtained by rotating R around the y-axis.

3. Suppose that the functions f and g are defined and positive on some interval $[a, \infty)$.

   - If $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$, we say that f grows more slowly than g or that g grows more rapidly than f , as $x \to \infty$. This may be indicated by writing $f << g$. (We use this terminology even if f and/or g do not in fact "grow" as $x \to \infty$; for example, they might go to zero).

   (a) Arrange the following eight functions in order of rate of growth, from least rapidly growing to most rapidly growing. indicating any cases in which the functions grow at comparable rates. Justify your answers. It is helpful to least check that if $f << g$ and $g << h$ then $f << h$.

   $$x; \ln x; \sqrt{x^3 + x}; e^x; \ln(\ln x); \ln(e^{x^2} + 1); x\ln(x); e^{x^2}.$$

   (b) Arrange the reciprocals of the same eight functions in order of rate of growth. (Hint: if f and g are related by $f << g$, what is the relation between $1/g$ and $1/f$?)

   (c) For which of the reciprocals $h(x) = 1/f(x)$ in (b) does the integral $\int_{100}^{\infty} h(x)dx$ converge? (Suggestion: Use the Comparison Theorem on page 536 of the text. To do so you will need to answer this queston: if $f << g$ is it true that $f(x) \leq g(x)$? Or true for some x?)