Math 152, Fall, 2004, Workshop 3

Honors Section

1. Calculate four of the following five indefinite integrals:

   (a) $\int x \cos x^2 dx$.

   (b) $\int x^2 \cos x dx$.

   (c) $\int x^2 \cos x^2 dx$.

   (d) $\int x^2 \cos x^3 dx$.

   (e) $\int x^3 \cos x^2 dx$.

2. A particle of mass 2 kilogram moves along a line; its position relative to the origin at time t second is $s(t) = t^3$ meters.

   (a) Use Newton's law ($F = ma$) to find the force $F = f(t)$ which acts on the body at time t, and then find the average value of this force between $t = 0$ sec and $t = 2$ sec. This is the time average $F_{\text{time average}}$ of the force. Draw a graph of the function $f(t)$ for $0 \leq t \leq 2$. Find the time t^* for which $f(t^*) = F_{\text{time average}}$ and give a graphical interpretation.

   (b) Find a formula $F = g(s)$ for the force which acts on the body when it is at position s, and then find the averge value of $g(s)$ between $s = s(0)$ and $s = s(2)$. This is the distance average $F_{\text{distance average}}$ of the force as the body moves from position $s(0)$ to position $s(2)$. Draw a graph of the function $g(s)$ for $s(0) \leq s \leq s(2)$. Find the position s^* for which $g(s^*) = F_{\text{distance average}}$ and give a graphical interpretation. (Notice that the time and distance averages of the force are different!)

3. (a) Obtain the reduction formula

$$\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx \qquad (*)$$

using integration by parts with $u = (\ln x)^n$ and $dv = dx$.

(b) Use the substitution $x = e^y$ to replace $(*)$ with a formula involving functions of y.

(c) Give a direct proof of the formula obtained in (b) using integration by parts.

4. In Section 2.1, average velocity was defined as

$$\text{average velocity} = \frac{\text{distance traveled}}{\text{time elapsed}}$$

(page 90). This was used to motivate a definition of *instantaneous velocity* as the derivative of distance with respect to time. Thereafter, the word "velocity" always referred to instantaneous velocity. We now have another meaning of the word "average," introduced in Section 6.5. Show that the old meaning of average velocity is the average of a function giving velocity as a function of time.

If the velocity is always of the same sign, the function giving distance as a function of time has an inverse, so all properties of the motion can be expressed as a function of distance. It is thus possible to consider averaging velocity with respect to distance. These two averages can be different. Show that this happenes when the motion is given by *free fall*, with displacement given by $s = \frac{1}{2}gt^2$.