Math 152, Fall, 2004, Workshop 2

Honors Section

1. Start with the region $\mathcal{A}$ in the first enclosed by the x-axis and the parabola $y = 2x(2 - x)$. Then obtain solids of revolution $\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{S}_3$ by rotating $\mathcal{A}$ about the line

$$y = 4, \qquad y = -2, \qquad \text{and} \qquad x = 4,$$

respectively. All three solids are (unusual) "doughnuts" which are 8 units across, whose holes are 4 units across, and whose heights are 2 units. Sketch them.

   (a) Which do you expect to have larger volume, $\mathcal{S}_1$ or $\mathcal{S}_2$? Compute their volume and check your guess.

   (b) Compute the volume of $\mathcal{S}_3$.

2. Consider the first quadrant region $\mathcal{A}$ bounded by the curve $y = x^2$, the tangent line to this curve at (1, 1), and the x-axis. Sketch this region.

   (a) Set up an integral giving the volume of the solid obtained by rotating $\mathcal{A}$ about the y-axis using the method of washers.

   (b) Set up an integral giving the volume of the solid obtained by rotating $\mathcal{A}$ about the y-axis using the method of cylindrical shells.

   (c) Compute one of the integrals above to find this volume.

   (d) Find the volume of the solid obtain by rotating $\mathcal{A}$ about x-axis. You may use any method.