Math 152, Fall, 2004, Workshop 10

Honors Section

1. (a) For each power series below, use the ratio test to determine all values of x for which the series converges absolutely, then analyze the behavior of the series at the endpoints in order to determine the interval of convergence.

$$(i)\ \sum_{n=0}^{\infty} \frac{nx^n}{n^2+1}, \qquad (ii)\ \sum_{n=1}^{\infty} \frac{n^2(x-1)^n}{2^n}, \qquad (iii)\ \sum_{n=1}^{\infty} \frac{3^n x^n}{n^2}.$$

   (b) Can you cook up a power series whose interval of convergence is the interval $(0,1]$, that is, the interval defined by $0 < x \leq 1$? How about $(0,1)$? Give an explicit series or explain why you can't.

2. Consider the function defined by

$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!}.$$

   (a) Write out the first five terms of the series (up to $n = 4$; remember that $0! = 1$).

   (b) Determine the interval of convergence; this is the domain of f.

   (c) Verify in the following two ways that

$$f'(x) = -2xf(x) : \tag{1}$$

      i. Compute both sides of (1) in terms of the five terms you wrote out in;

      ii. Verify (1) using the original summation (sigma) notation. This is a bit tricky (which is why it is a good idea to do i first); you will have to make a change of summation index.

   (d) Explain why $y = f(x)$ is a solution of the initial value problem

$$y' = 2xy, \qquad y(0) = 1.$$

(e) Solve this initial value problem and thereby obtain a formula for f(x) in terms of "elementary functions" (those found on your calculator).

3. Keep the notation of the previous problem, and let

$$s_N(x) = \sum_{n=1}^{N} (-1)^n \frac{x^{2n}}{n!},$$

e.g., $s_3(x) = 1 - x^2 + x^4/2 - x^6/6$.

(a) Using the formula you discovered for $f(x)$, use your calculator to graph f and the partial sums s_0, s_2, s_4, and s_6 at the same time, in a window where $0 \leq x \leq 1.2$. Copy your graph to your workshop, identifying the different curves. Does it appear that s_N becomes a better approximation to f as N increases?

(b) Use the alternating series error formula to obtain an upper bound for the error in the approximation $f(x) \simeq s_6(x)$, $0 \leq x \leq 1.2$. Your answer should be a single number that applies to all x in the range $0 \leq x \leq 1.2$. Explain why this number is consistent with the graph from (a).