Numerarical Information

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Differentiation Formulas

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

Integration Formulas

$$\begin{aligned} \int \sec x dx &= \ln|\sec x + \tan x| + C \\ \int \csc x dx &= \ln|\csc x - \cot x| + C \\ &= -\ln|\csc x + \cot x| + C \end{aligned}$$

Trigonometric Identitties

$$\begin{aligned} 1 &= \sin^2\theta + \cos^2\theta \\ \sec^2\theta &= 1 + \tan^2\theta \\ \sin(2\theta) &= 2\sin\theta\cos\theta \\ \cos\theta &= \cos^2\theta - \sin^2\theta \\ \sin^2\theta &= \frac{1}{2}(1-\cos 2\theta) \\ \cos^2\theta &= \frac{1}{2}(1+\cos 2\theta) \\ \sin x\cos y &= \frac{1}{2}(\sin(x-y)+\sin(x+y)) \\ \sin x\sin y &= \frac{1}{2}(\cos(x-y)-\cos(x+y)) \\ \cos x\cos y &= \frac{1}{2}(\cos(x-y)+\cos(x+y)) \\ \sin(x\pm y) &= \sin x\cos y \pm \cos x\sin y \\ \cos(x\pm y) &= \cos x\cos y \mp \sin x\sin y \end{aligned}$$

Midpoint Rule

$$\begin{aligned} \int_a^b f(x)dx &\simeq M_n \\ &= \Delta x(f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n)) \end{aligned}$$

$$(\Delta x = \frac{b-a}{n}, x_i = a + i\Delta x, \overline{x}_i = \frac{x_{i-1}+x_i}{2})$$

Suppose $|f''(x)| \le K$ for $a \le x \le b$. If E_M is the error in the middle point rule, then

$$|E_M| \le \frac{K(b-a)^3}{24n^2}.$$

Trapezoidal Rule

$$\begin{aligned} \int_a^b f(x)dx &\simeq T_n \\ &= \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + \cdots \\ &\qquad + 2f(x_{n-1}) + f(x_n)) \end{aligned}$$

$$(\Delta x = \frac{b-a}{n}, x_i = a + i\Delta x)$$

Suppose $|f''(x)| \le K$ for $a \le x \le b$. If E_T is the error in the middle point rule, then

$$|E_T| \le \frac{K(b-a)^3}{12n^2}.$$

Simpson's Rule

$$\begin{aligned} \int_a^b f(x)dx &\simeq S_n \\ &= \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots \\ &\qquad + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)) \end{aligned}$$

$$(n \text{ even}, \Delta x = \frac{b-a}{n}, x_i = a + i\Delta x)$$

Suppose $|f^{(4)}(x)| \le K$ for $a \le x \le b$. If E_S is the error in the middle point rule, then

$$|E_S| \le \frac{K(b-a)^5}{180n^4}.$$

Length of a Plane Curve

The length of the curve $y = f(x)$ for $a \le x \le b$ is $\int_a^b \sqrt{1+(f'(x))^2}dx$.

The length of the curve $x = g(y)$ for $c \le y \le d$ is $\int_c^d \sqrt{1+(g'(y))^2}dy$.

The length of the parametric curve $x = x(t)$, $y = y(t)$ for $a \le t \le b$ is $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}dt$.

Area of a Surface of Revolution

Then the area of the surface generated by rotating the curve $y = f(x)$ for $a \le x \le b$ about the x-axis is $\int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}dx$.

The area of the surface generated by rotating the curve $x = g(y)$ for $c \le y \le d$ about the y-axis is $\int_c^d 2\pi g(y)\sqrt{1+(g'(y))^2}dy$.