Frequently used Maclaurin series

$$
\begin{aligned}
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} && |x| < \infty \\
\sin x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} && |x| < \infty \\
\cos x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} && |x| < \infty \\
\ln(1+x) &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} && -1 < x \le 1 \\
\tan^{-1} x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} && |x| \le 1 \\
(1+x)^m &= \sum_{n=0}^{\infty} \binom{m}{n} x^n && |x| < 1
\end{aligned}
$$

where

$$
\binom{m}{n} = \begin{cases} \dfrac{m(m-1)(m-2)\cdots(m-n+1)}{n!} & n > 0 \\ 0 & n = 0 \end{cases}
$$

Taylor's inequality

Let $f(x)$ be $(n+1)$-times differentiable such that $|f^{(n+1)}(x)| \le M$ when $|x-a| \le d$. Then the remainder

$$
R_n(x) = f(x) - T_n(x) = f(x) - \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k
$$

of the Taylor's series satisfies the inequality

$$
|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1}
$$

for $|x-a| \le d$.

A few other Taylor series

Each of the following series has radius of convergence $\frac{\pi}{2}$:

$$
\begin{aligned}
\tan x &= x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + \cdots \\
\sec x &= 1 + \frac{1}{2}x^2 + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8 + \cdots \\
\cot &= -\left(x - \frac{\pi}{2}\right) - \frac{1}{3}\left(x - \frac{\pi}{2}\right)^3 - \frac{2}{15}\left(x - \frac{\pi}{2}\right)^5 - \frac{17}{315}\left(x - \frac{\pi}{2}\right)^7 - \cdots \\
\csc x &= 1 + \frac{1}{2}\left(x - \frac{\pi}{2}\right)^2 + \frac{5}{24}\left(x - \frac{\pi}{2}\right)^4 + \frac{61}{720}\left(x - \frac{\pi}{2}\right)^6 + \cdots
\end{aligned}
$$