Answers to Part 3 of the review problems for final exam, Math 151, Sections 13, 14, 15

1.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(\frac{i}{n}\right)^2+\frac{2i}{n}\right]\frac{1}{n}=\int_0^1(x^2+2x)dx=\frac{4}{3}.$$

2. (a) $\displaystyle\int_5^6\frac{dt}{(t-4)^2}=\frac{1}{2}.$

(b) $\displaystyle\int\frac{e^{\sqrt{x}}}{\sqrt{x}}dx=2e^{\sqrt{x}}+C.$

(c) $\displaystyle\int x^2\sqrt{1+x^3}dx=\frac{2}{9}(1+x^3)^{\frac{3}{2}}+C.$

(d) $\displaystyle\int_1^9\frac{\sqrt{x}-2x^2}{x}dx=-76.$

(e) $\displaystyle\int(3e^x+7\sec^2 x+5(1-x^2)^{-\frac{1}{2}})dx=3e^x+7\tan x+5\sin^{-1}x+C.$

(f) $\displaystyle\int_{\frac{4}{\pi}}^{\frac{3}{\pi}}\frac{1}{x^2}\sec^2\left(\frac{1}{x}\right)dx=-\tan\left(\frac{\pi}{3}\right)+\tan\left(\frac{\pi}{4}\right)=1-\sqrt{3}.$

3.

$$\int_{-\pi}^{\pi}\frac{x^5+\tan x}{1+2x^2}dx=0$$

since $\displaystyle\frac{x^5+\tan x}{1+2x^2}$ is an odd function.

4. (a) $\displaystyle\frac{d}{dx}\int_1^{10x}\ln(t^4+3t^2+7)dt=10\ln(10000x^4+300x^2+7).$

(b) $\displaystyle\frac{d}{dx}\int_1^{x^2}\sin(\cos(\sqrt{t}))dt=2x\sin(\cos(\sqrt{x^2})).$

5. Find the areas bounded by the following curves:

(a) $\displaystyle A=\int_0^4(8x-2x^2)dx=\frac{64}{3}.$

(b) $\displaystyle A=\int_0^{\frac{\pi}{4}}(\cos x-\sin x)dx=\sqrt{2}-1.$