Review problems for Exam #2, Math 151, Sections 13, 14, 15

1. Find the derivatives of the following functions:

   i) $\ln(\sin(5x) + e^{2x})$. ii) $\tan^{-1}(3x^2 - 5x + 2)$. iii) $e^{\cos x}\sin(e^x)$.

2. Find the line tangent to the curve defined by the equation

$$\ln(xy) + 2x - y + 1 = 0$$

   at the point $(\frac{1}{2}, 2)$.

3. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $f(4)$ possibly be?

4. Suppose that $f(x)$ is a differentiable function with $f(2) = 3$ and $f'(x) = \sqrt{23 - 7x}$ for all real numbers x. Suppose also that $g(x) = 5x^2 - 3$. Let $h(x) = f(g(x))$.

   (a) Compute $h(1)$. Find $h'(x)$ (only in terms of x) and then calculate $h'(1)$.

   (b) Use your answer above and linearization to find an approximation value of $h(0.95)$. (You need not simplify the answer!)

   (c) It is true that $h''(1) = -\frac{260}{3}$. Is the approximation you found above likely to be greater than or less than the true value of $h(0.95)$? Give a brief reason supporting your answer.

5. (a) Find the linear approximation of the function $f(x) = \sqrt{1-x}$ at $a = 0$.

   (b) Use the approximation determined to approximate the number $\sqrt{0.99}$.

6. Find the following limits which could be a real number or ∞ or $-\infty$:

   i) $\lim\limits_{x\to\infty} \dfrac{\ln(\ln(x))}{(\ln(x))^2}$. ii) $\lim\limits_{x\to 0} \dfrac{x^2}{1-\cos(\pi x)}$. iii) $\lim\limits_{x\to 0} \dfrac{\sin(x) - x}{(e^x - 1)^2}$.

   iv) $\lim\limits_{x\to\infty}(x - \sqrt{x^2 - 3x})$. v) $\lim\limits_{x\to 0}(1+2x)^{\frac{1}{x}}$.

7. Let $f(x) = 1 - x - \sin(3x)$.

   (a) Show that $f(x)$ has a root in $[0, 1]$.

   (b) If Newton's method is used to find a root of $f(x)$, we obtain a sequence of numbers given by a formula of the form $x_{n+1} = g(x_n)$. What is the function $g(x)$?

   (c) If the starting guess for Newton's method is taken to be $x_0 = 0.5$, find the next two approximations x_1 and x_2.

8. Let $f(x) = (x^2 - 1)e^x$.

(a) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

(b) Compute $f'(x)$. Find the exact solutions of $f'(x) = 0$ and find where $f'(x)$ is positive and where it is negative.

(c) Compute $f''(x)$. Find the exact solutions of $f''(x) = 0$ and find where $f''(x)$ is positive and where it is negative.

(d) Use the information above and the analytic tools of calculus to sketch the graph of $f(x)$.

9. Suppose that $f(x)$ is a function for all x. Assume also that $f'(x)$ and $f''(x)$ exist for all x (which means, in particular, that the graph of $f(x)$ has no corner). Assume that:

$$f(0) = -2, f(2) = 2, f(4) = 1,$$
$$f'(0) = f'(2) = f'(4) = 0,$$
$$f'(x) < 0 \text{ for } x < 0 \text{ and } 2 < x < 4,$$
$$f'(x) > 0 \text{ for } 0 < x < 2,$$
$$f''(x) < 0 \text{ for } x < -1 \text{ and } 1 < x < 3,$$
$$f''(x) > 0 \text{ for } -1 < x < 1 \text{ and } 3 < x.$$

(a) Find all local minimum and maximum inside $(-\infty, 3)$.

(b) Find all inflection points.

(c) Sketch the part of the graph of $f(x)$ over the interval $(-\infty, 4]$.

(d) Is it possible that $f'(x) < 0$ for all $x > 4$? Explain your anwers.

10. Ant A is crawling up a vertical pole at 0.3 meters/minute. At the same time ant B is crawling away from the base of the pole on the horizontal ground at 0.4 meters/minute.

(a) If θ is the angle that ant B sees between the base of the pole and ant A, a is the distance from ant A to the base of the pole and b is the distance from ant B to the base of the pole, express θ as a function of a and b.

(b) Compute θ and $\frac{d\theta}{dt}$ at the instant that ant A is 10 meters up the pole and ant B is 5 meters from the base of the pole.

11. A farmer with 450 feet of fencing wants to enclose the four sides of a rectangular region and then divide the region into four pens with fencing parallel to one side of the rectrangle. What is the largest possible total area of the four pens?

12. Suppose that $f''(x) = -12x + \cos x$, $f(\pi) = 2$ and $f'(\pi) = -1$. What is $f(\frac{\pi}{6})$?