Answers to review problems for Exam #2, Math 151, Sections 13, 14, 15

1. i) $\dfrac{5\cos(5x)+2e^{2x}}{\sin(5x)+e^{2x}}$. **Hint:** Use the chain rule.

   ii) $\dfrac{6x-5}{1+(3x^2-5x+2)^2}$. **Hint:** Use the chain rule.

   iii) $-e^{\cos x}\sin x\sin(e^x)+e^{\cos x}\cos(e^x)e^x$. **Hint:** Use the product rule and the chain rule.

2. $y=8x-2$. **Hint:** Use the implicit differentiation to find the slope.

3. $f(4)\geq 16$. **Hint:** Use the mean value theorem: $f(4)-f(1)=f'(c)(4-1$ for some c satisfying $1\leq c\leq 4$. Since $f'(c)\geq 2$, $f'(c)(4-1)\geq 6$. So $f(4)\geq 16$.

4. (a) $h'(x)=10x\sqrt{44-35x^2}$ and $h'(1)=30$. **Hint:** Use the chain rule.

   (b) $h(0.95)\simeq 1.5$. **Hint:** Use $h(0.95)\simeq h(1)+h'(1)(0.95-1)$, $h(1)=f(2)=3$ and $h'(1)=30$.

   (c) Likely to be greater than the true value of $h(0.95)$. Reason: Since $h''(1)=-\frac{260}{3}$ is negative, the graph of $h(x)$ is concave downward near $x=1$. So the tangent line is above the graph. Thus the linear approximation is greater than the true value.

5. (a) $1-\frac{x}{2}$. **Hint:** Use the linear approximation formula: $f(x)\simeq f(a)+f'(a)(x-a)$.

   (b) 0.995. **Hint:** Take x to be 0.01 in the linear approximation above.

6. i) 0. **Hint:** Use L'Hôpital's rule.

   ii) $\dfrac{2}{\pi^2}$. **Hint:** Use L'Hôpital's rule.

   iii) 0. **Hint:** Use L'Hôpital's rule.

   iv) $\dfrac{3}{2}$. **Hint:** Use $x-\sqrt{x^2-3x}=\dfrac{3x}{x+\sqrt{x^2-3x}}$ and then use either the method of compute limits of algebraic functions or L'Hôpital's rule.

   v) e^2. **Hint:** Take the logarithm of $(1+2x)^{\frac{1}{x}}$ and then use L'Hôpital's rule.

7. (a) We have $f(0)=1$ and $f(1)=-\sin 3$. Since $0<3<\pi$, we know that $\sin 3>0$. So $f(1)=-\sin 3<0$. Thus 0 is between $f(0)$ and $f(1)$. By the intermediate value theorem, there is c between 0 and 1 such that $f(c)=0$, that is, $f(x)$ has a root in $[0,1]$.

   (b) $g(x)=x+\dfrac{1-x-\sin(3x)}{1+3\cos(3x)}$. **Hint:** Use the formula $x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}$.

(c) $x_1 = 0.5 + \dfrac{0.5 - \sin(1.5)}{1 + 3\cos(1.5)}$ and

$$x_2 = 0.5 + \frac{0.5 - \sin(1.5)}{1 + 3\cos(1.5)} + \frac{0.5 - \frac{0.5-\sin(1.5)}{1+3\cos(1.5)} - \sin(1.5 + 3\frac{0.5-\sin(1.5)}{1+3\cos(1.5)})}{1 + 3\cos(1.5 + 3\frac{0.5-\sin(1.5)}{1+3\cos(1.5)})}.$$

8. (a) $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = 0$. **Hint:** Since $\lim_{x\to\infty}(x^2 - 1) = \infty$ and $\lim_{x\to\infty} e^x = \infty$, $\lim_{x\to\infty} f(x) = \infty$. For $\lim_{x\to-\infty} f(x)$, use L'Hôpital's rule.
   (b) $f'(x) = (x^2 + 2x - 1)e^x$. The exact solutions of $f'(x) = 0$ are $-1 \pm \sqrt{2}$. When $x \le -1 - \sqrt{2}$ or $x > -1 + \sqrt{2}$, $f'(x) > 0$. When $-1 - \sqrt{2} < x < -1 + \sqrt{2}$, $f'(x) < 0$.
   (c) $f''(x) = (x^2 + 4x + 1)e^x$. The exact solutions of $f''(x) = 0$ are $-2 \pm \sqrt{3}$. When $x \le -2 - \sqrt{3}$ or $x > -2 + \sqrt{3}$, $f''(x) > 0$. When $-2 - \sqrt{3} < x < -2 + \sqrt{3}$, $f''(x) < 0$.
   (d) See the picture.

9. (a) Local minimum: 0. Local maximum: 2. **Hint:** Use the information on $f'(x)$.
   (b) Inflection points: $-1, 1, 3$. **Hint:** Use the information on $f''(x)$.
   (c) See the picture.
   (d) No. For $x > 4$, $f''(x) > 0$. So $f'(x)$ is increasing when $x > 4$. But $f'(4) = 0$. So $f'(x) \ge 0$ for $x > 4$.

10. (a) $\theta = \tan^{-1}\frac{a}{b}$. **Hint:** Draw a picture.
    (b) $\theta = \tan^{-1} 2$ and $\frac{d\theta}{dt} = -\frac{1}{50}$. **Hint:** θ can be obtained by just substitute the values of a and b. $\frac{d\theta}{dt}$ is obtained by using chain rules (both a and b are functions of t) and then using $a = 10$, $b = 5$, $a' = 0.3$ and $b' = 0.4$.

11. 5062.5. **Hint:** Let y be the length of the sides with parallel fencing inside and x the length of the other two sides. Then the area $A = xy$ and we have the constraint $2x + 5y = 450$. So $A = 90x - \frac{2}{5}x^2$. The domain is $[0, 225]$. A reaches its maximum at $x = 112.5$. Then calculate the area at this x.

12. $f(\dfrac{\pi}{6}) = -\dfrac{325\pi^3}{108} + \dfrac{5\pi}{6} - \dfrac{\sqrt{3}}{2} + 1$. **Hint:** Find $f'(x) = -6x^2 + \sin x + 6\pi^2 - 1$ first. Then find $f(x) = -2x^3 - \cos x + (6\pi^2 - 1)x + 1 - 4\pi^3 + \pi$.