Review problems for Exam #1, Math 151, Sections 13, 14, 15

1. (a) Find the domains and ranges of the following functions:

   i) $f(x) = \frac{1}{\sqrt{x-5}+2}$. ii) $g(x) = \frac{1}{x^2-9}$. iii) $h(x) = -2\log(x+5) - 7$.

   (b) Which functions have inverses?

   (c) For functions having inverses, find their inverse functions and state what are the domains and ranges of these inverse functions.

2. Determine whether the following limits exist, find the exact values of the limits if the limits exist and indicate which limits approach infinity, if any:

   i) $\lim_{x\to 3} \frac{(x+3)(x-5)}{x^2-9}$. ii) $\lim_{x\to 0} \frac{(x+4)(x-5)}{x^2-9}$. iii) $\lim_{x\to 3} \frac{x^3\sin(x-3)+x^2-9}{x-3}$.

   iv) $\lim_{x\to 1} \sin(e^{x-1})$. v) $\lim_{x\to 0} \ln(2-\cos x)$. vi) $\lim_{x\to 0} \frac{x}{\sin x}$. vii) $\lim_{x\to 0} \cos\frac{1}{x}$.

   ix) $\lim_{x\to\infty} \frac{8x^3-3x^2+x+6}{4x^3+15x^2-7x-2}$. x) $\lim_{x\to\infty} \frac{8x^4-3x^2+x+6}{4x^3+15x^2-7x-2}$.

3. Let

$$f(x) = \begin{cases} x^2+1, & x < 0 \\ 4x+1, & 0 \le x < 2 \\ x^2+1, & x \ge 2, \end{cases}$$

   (a) Calculate $\lim_{x\to 0^-} f(x)$, $\lim_{x\to 0^+} f(x)$, $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^-} f(x)$.

   (b) Find the x-values (if any) at which f is not continuous.

4. Using the definition of derivative, find $f'(x)$ for

$$\text{i) } f(x) = \frac{3}{\sqrt{x+1}}. \qquad \text{ii) } f(x) = \frac{4}{2x+5}.$$

5. Let $f(x) = \frac{x^2}{x^2-4}$.

   (a) Find all the vertical and horizontal asymptotes of $f(x)$.

   (b) Find all x-values at which the tangent lines of the graph of the function $y = f(x)$ are horizontal.

   (c) Find the equation of the tangent line to the graph of $y = f(x)$ at the point $(1, -\frac{1}{3})$.

6. Find $f'(x)$ if

   (a) $f(x) = \sqrt{3x}$.

   (b) $f(x) = \dfrac{7}{x^{2/7}} + \sqrt{x^3} + e$.

   (c) $f(x) = \dfrac{1}{(3x+1)^2}$.

   (d) $f(x) = e^x \sin x$.

   (e) $f(x) = x^2 \sin x \tan x$.

   (f) $f(x) = \dfrac{x-3}{x+3}$.

   (g) $f(x) = 7x \cos x + 4x^4 e^x - x^3$.

   (h) $f(x) = \dfrac{\cos x}{e^x + x}$.

7. Is there a solution of the equation $3x^3 - 9x^2 + 2x - 3 = 0$ between 0 and 3?

8. Suppose that $f(x)$ and $g(x)$ are differentiable functions such that

$$f(4) = -1, \quad f'(4) = 2, \quad g(4) = -3, \quad g'(4) = 5.$$

   Calculate the the derivatives of the following functions at $x = 4$:

$$A(x) = 3f(x) + 2g(x); \qquad B(x) = 3f(x)g(x).$$