Review problems for Exam #1, Math 151, Sections 13, 14, 15

1. (a) Find the domains and ranges of the following functions:

i)
$$f(x) = \frac{1}{\sqrt{x-5+2}}$$
. ii) $g(x) = \frac{1}{x^2-9}$. iii) $h(x) = -2\ln(x+5) - 7$.

- (b) Which functions have inverses?
- (c) For functions having inverses, find their inverse functions and state what are the domains and ranges of these inverse functions.
- 2. Determine whether the following limits exist, find the exact values of the limits if the limits exist and indicate which limits approach infinity, if any:

$$\begin{array}{l} \text{i)} \lim_{x \to 3} \frac{(x+3)(x-5)}{x^2 - 9}, & \text{ii)} \lim_{x \to 0} \frac{(x+4)(x-5)}{x^2 - 9}, & \text{iii)} \lim_{x \to 3} \frac{x^3 \sin(x-3) + x^2 - 9}{x - 3}, \\ \text{iv)} \lim_{x \to 1} \sin(e^{x-1}), & \text{v)} \lim_{x \to 0} \ln(2 - \cos x), & \text{vi)} \lim_{x \to 0} \frac{x}{\sin x}, & \text{vii)} \lim_{x \to 0} \cos \frac{1}{x}, \\ \text{ix)} \lim_{x \to \infty} \frac{8x^3 - 3x^2 + x + 6}{4x^3 + 15x^2 - 7x - 2}, & \text{x)} \lim_{x \to \infty} \frac{8x^4 - 3x^2 + x + 6}{4x^3 + 15x^2 - 7x - 2}. \\ 3. \text{ Let} \\ f(x) = \begin{cases} x^2 + 1, & x < 0\\ 4x + 1, & 0 \le x < 2\\ x^2 + 1, & x \ge 2, \end{cases} \end{cases}$$

- (a) Calculate $\lim_{x \to 0^{-}} f(x)$, $\lim_{x \to 0^{+}} f(x)$, $\lim_{x \to 2^{-}} f(x)$ and $\lim_{x \to 2^{+}} f(x)$.
- (b) Find the x-values (if any) at which f is not continuous.
- 4. Using the definition of derivative, find f'(x) for

i)
$$f(x) = \frac{3}{\sqrt{x+1}}$$
. ii) $f(x) = \frac{4}{2x+5}$.

5. Let $f(x) = \frac{x^2}{x^2 - 4}$.

- (a) Find all the vertical and horizontal asymptotes of f(x).
- (b) Find all x-values at which the tangent lines of the graph of the function y = f(x) are horizontal.
- (c) Find the equation of the tangent line to the graph of y = f(x) at the point $(1, -\frac{1}{3})$.

6. Find f'(x) if

- (a) $f(x) = \sqrt{3x}$. (b) $f(x) = \frac{7}{x^{2/7}} + \sqrt{x^3} + e$. (c) $f(x) = \frac{1}{(3x+1)^2}$. (d) $f(x) = e^x \sin x$. (e) $f(x) = x^2 \sin x \tan x$. (f) $f(x) = \frac{x-3}{x+3}$. (g) $f(x) = 7x \cos x + 4x^4 e^x - x^3$. (h) $f(x) = \frac{\cos x}{e^x + x}$.
- 7. Is there a solution of the equation $3x^3 9x^2 + 2x 3 = 0$ between 0 and 3?
- 8. Suppose that f(x) and g(x) are differentiable functions such that

$$f(4) = -1$$
, $f'(4) = 2$, $g(4) = -3$, $g'(4) = 5$.

Calculate the derivatives of the following functions at x = 4:

$$A(x) = 3f(x) + 2g(x);$$
 $B(x) = 3f(x)g(x).$