Answers to Review problems for Exam #1, Math 151, Sections 13, 14, 15

1. (a) i) Domain: $[5, \infty)$. Range: $(0, \frac{1}{2}]$.
   ii) Domain: $(\infty, -3) \cup (-3, 3) \cup (3, \infty)$. Range: $(-\infty, \infty)$.
   iii) Domain: $(-5, \infty)$. Range: $(-\infty, \infty)$.
   (b) $f(x)$ and $h(x)$.
   (c) $f^{-1}(x) = \left(\frac{1}{x} - 2\right)^2 + 5$. Donain: $(0, \frac{1}{2}]$. Range: $[5, \infty)$.
   $h^{-1}(x) = e^{-\frac{x+7}{2}} - 5$. Domain: $(-\infty, \infty)$. Range: $(-5, \infty)$.

2. i) The limit does not exist. ii) $\frac{20}{9}$. iii) 33.
   iv) $\sin 1$. v) 0. vi) 1. vii) The limit does not exist.
   ix) 2. x) ∞.

3. (a) $\lim_{x \to 0^-} f(x) = 1$, $\lim_{x \to 0^+} f(x) = 1$, $\lim_{x \to 2^-} f(x) = 9$ and $\lim_{x \to 2^+} f(x) = 5$.
   (b) $x = 2$.

4. i) $f'(x) = \frac{-3}{2(\sqrt{x+1})^3}$. Hint: Use

$$(\sqrt{x+1} - \sqrt{x+h+1})(\sqrt{x+1} + \sqrt{x+h+1}) = (x+1) - (x+h+1).$$

   ii) $f'(x) = -\frac{8}{(2x+5)^2}$.

5. Let $f(x) = \frac{x^2}{x^2 - 4}$.
   (a) Vertical asymptotes: $x = 2$ and $x = -2$. Horizontal asymptote: $y = 1$.
   (b) $x = 0$.
   (c) $y = -\frac{8}{9}x + \frac{5}{9}$.

6. (a) $f'(x) = \frac{\sqrt{3}}{2\sqrt{x}}$.
   (b) $f'(x) = -2x^{-\frac{9}{2}} + \frac{3}{2}x^{\frac{1}{2}}$.
   (c) $f'(x) = \frac{-6}{(3x+1)^3}$.
   (d) $f'(x) = e^x \sin x + e^x \cos x$.

(e) $f'(x) = 2x \sin x \tan x + x^2 \sin x + x^2 \sin x \sec^2 x$.

(f) $f'(x) = \dfrac{6}{(x+3)^2}$.

(g) $f'(x) = 7\cos x - 7x \sin x + 16x^3 e^x + 4x^4 e^x - 3x^2$.

(h) $f'(x) = \dfrac{-e^x \sin x - x \sin x - e^x \cos x - \cos x}{(e^x + x)^2}$.

7. Yes. Hint: Use the intermediate value theorem.

8. $A'(4) = 15$. $B'(4) = -33$.