Workshop 8

1. Consider a right circular cone of base radius R and height H placed up-side down such that its tip is at the origin and its axis coincides with the $z$axis. Describe the cone in all the 3 coordinate systems; namely, rectangular, cylindrical and spherical.

(For example, a sphere with radius R with center at the origin is described in rectangular coordinates as $x^2 + y^2 + z^2 = R^2$ and in spherical coordinates as $\rho = R$.)

2. A particle P moves in the plane. The *Rectangular Observer* computes the magnitude of the speed and acceleration of P by observing the x- and y-coordinates of P (as functions of time) and using these formulas: speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ and acceleration $= \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2}$.

Another observer, the *Polar Observer*, finds it easier and more natural to measure the polar coordinates r and θ of P (as functions of time), using herself as the origin, of course. The *Polar Observer* computes $\frac{dr}{dt}$, $\frac{d\theta}{dt}$, $\frac{d^2r}{dt^2}$, etc. What formula should the *Polar Observer* use to compute the speed of the particle P? Deduce your answer from the formula for speed displayed above and the relationships among x, y, r, and θ. Finally, if r and $\frac{d\theta}{dt}$ are constant, derive the following formula, valid for uniform circular motion: acceleration $= (\text{speed})^2/r$.