Question:

The linear approximation for the function $f(x) = x^5$ near $x = 2$ is $32 + 80(x - 2)$. (You should check this!)

a) What number a will give a good quadratic approximation $x^5 \approx 32 + 80(x - 2) + a(x - 2)^2$ near $x = 2$?

b) If this approximation is used for various x's in the interval $[2, 2.1]$, can you be certain that the error is no bigger than .05? Explain, using Taylor's inequality (the *Error Bound*).

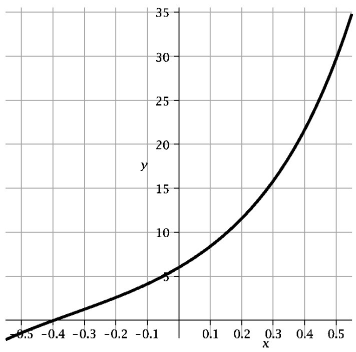
c) Graph $x^5 - \left(32 + 80(x - 2) + a(x - 2)^2\right)$ (using the value of a previously found) in the interval $[2, 2.1]$.

Question:

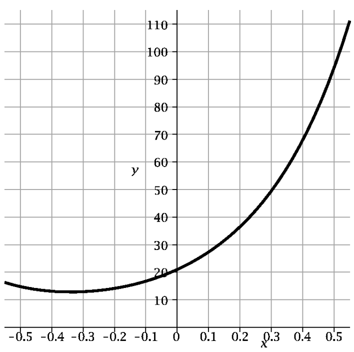
Suppose $f(x) = e^{x^2+\sin x}$. Here are values of f and some of its derivatives at 0:

$$f(0) = 1;\ \ f'(0) = 1;\ \ f''(0) = 3;\ \ f^{(3)}(0) = 6;\ \ f^{(4)}(0) = 21;\ \ f^{(5)}(0) = 52\,.$$

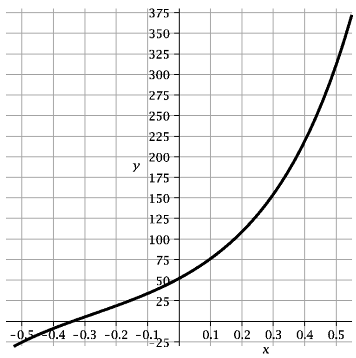
Below are graphs of $f^{(3)}(x)$, $f^{(4)}(x)$, and $f^{(5)}(x)$ on the interval $[-.5, .5]$.



Graph of $f^{(3)}(x)$ *on* $[-.5, .5]$



Graph of $f^{(4)}(x)$ *on* $[-.5, .5]$



Graph of $f^{(5)}(x)$ *on* $[-.5, .5]$

Assume this information is correct. No additional computation of the values of f or any of its derivatives is needed for this problem.

a) What is the second degree Taylor polynomial centered at 0 of f? *Do no unnecessary arithmetic!*

b) Find a polynomial $P(x)$ so that $|P(x) - f(x)| < .01$ for all x in the interval $\left[-\frac{1}{4}, \frac{1}{4}\right]$. You should write the polynomial and explain why the error is less than $.01 = \frac{1}{100}$.