Workshop 3

1. a) Suppose that m and n are integers. Compute $\displaystyle\int_0^{2\pi} (\cos(mx))\,(\cos(nx))\,dx$.

(Be careful: there will be two different results, one when $m = n$ and one when $m \neq n$.)

b) Suppose $f(x) = A\cos(x) + B\cos(2x) + C\cos(3x)$, and that you also know

$$\int_0^{2\pi} f(x)\cos(x)\,dx = 5\,; \quad \int_0^{2\pi} f(x)\cos(2x)\,dx = 6\,; \quad \int_0^{2\pi} f(x)\cos(3x)\,dx = 7\,.$$

Find A, B and C.

Note The ideas of this computation are used often with Fourier series, a standard method of analyzing periodic phenomena.

2. Some antiderivatives can be computed using "rationalizing substitutions" to change the integrals into integrals of rational functions which then can be computed using partial fractions. Here are some examples.

(1) $\displaystyle\int \frac{1}{x + 3\sqrt{x} + 2}\,dx$ (You may try $t = \sqrt{x}$.)

(2) $\displaystyle\int \frac{e^x + 1}{e^{2x} + 1}\,dx$

(3) $\displaystyle\int \frac{\sec^2\theta}{\tan^2\theta - 1}\,d\theta$

(4) $\displaystyle\int \frac{\sqrt{x}\,dx}{x^3 + 1}$ (If $\sqrt{x}\,dx = du$, then what is u?)

3. a) Compute $\displaystyle\int_1^2 \frac{dx}{x^2}$.

b) Compute $\displaystyle\int_1^2 \frac{dx}{x(x-m)}$ if m is a small positive number. What happens when $m \to 0^+$?

c) Compute $\displaystyle\int_1^2 \frac{1}{x^2+n}\,dx$ if n is a small positive number. What happens when $n \to 0^+$?

d) Sketch a graph of $\dfrac{1}{x^2}$, $\dfrac{1}{x(x-m)}$, and $\dfrac{1}{x^2+n}$ if $m = n = 0.1$ for x between 1 and 2.

4. An oil tank has the shape of a cylinder whose diameter is 4 feet. It is mounted so that the axis of the cylinder is horizontal (the circular cross-sections of the cylinder are vertical). If the depth of the water is 3 feet, what percentage of the total capacity of the tank is filled?

After drawing a picture and setting up this problem, solve it two ways:

i) Use elementary geometry (compare areas of circular sectors).

ii) Express the answer in terms of a definite integral, evaluate the integral exactly in terms of elementary functions using a trig substitution, and then obtain approximate numerial values for these functions using your calculator.