Workshop 2

1. Start with the region A in the first quadrant enclosed by the x-axis and the parabola $y = 2x(2-x)$. Then obtain solids of revolution S_1, S_2, and S_3 by revolving A about the lines $y = 4$, $y = -2$, and $x = 4$ respectively. All three solids are (unusual) "doughnuts" which are 8 units across, whose hole is 4 units across, and whose height is 2 units. Sketch them.

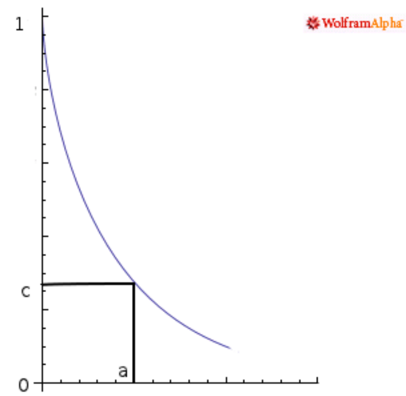
(a) Which do you expect to have larger volume, S_1 or S_2? Compute their volumes exactly and check your guess.

(b) Compute the volume of S_3. (It may be harder to guess in advance how S_3 compares in volume to S_2 and S_1.)

2. The curve $y = f(x)$ called *Tractrix*, has the following property: the derivative at any point x satisfies the formula:

$$f'(x) = \frac{-f(x)}{\sqrt{1 - f(x)^2}}.$$

Consider the region under the Tractrix, within the limits $0 \le x \le a$. Find the volume of the solid obtained by revolving this region around the $x-$axis in terms of the constant $c = f(a)$. [*Hint:* use $u-$substitution.]



3. Harry, an actual potter, knows fair bit of math. He loves to create pots of various shapes and sizes. Once day, he makes a pot by revolving some amount of clay on his wheel. Now he wants to find out the total mass of the pot by using integrals. However, there is one difficulty. Due to the rotation of the wheel, the clay now has a non-uniform density. He finds the following mathematical facts about his artifact:

(1) The solid could be obtained by revolving a positive function $f(x)$ over the interval $[a, b]$ (where a,b are positive numbers) about the $y-$axis.

(2) Density of clay varies as a function of the distance from the axis of rotation. At a distance d from the axis of rotation, it is given by $\rho(d)$ gms/units^3

Set-up the integral that would help Harry calculate the total mass of clay used. How would this integral change if the axis of rotation is changed to $x = -1$?