640:152 Calculus II, Spring 2012
Exam #2 Part I Questions

A typical 80-minute midterm consists of 2 "Part I" questions and 4-7 "Part II" questions, 12-16 points each. Answers given without any explanation or justification (words, phrases/sentences, and algebraic steps) may be given minimal credit.

PART I - LIMITS & INTEGRALS
Two questions similar to the types listed below will be chosen

1. Determine the limits
$$\lim_{n\to\infty} n^{\frac{1}{n}} \qquad \lim_{n\to\infty} c^{\frac{1}{n}} \qquad \lim_{n\to\infty} (1+\frac{1}{n})^n \qquad \lim_{n\to\infty} \frac{\ln n}{n} \qquad \lim_{n\to\infty} \sqrt{n^2+1}-n$$

2. Determine the limits
$$\lim_{x\to\infty} \sin\frac{1}{x} \qquad \lim_{x\to\infty} \frac{x^n}{e^x} \qquad \lim_{x\to\infty} \arctan x \qquad \lim_{x\to\infty} \frac{3x^4+5x^2+2x+17}{6x^4+8x+11} \qquad \lim_{x\to\infty} \frac{|x|+x}{1+x}$$

3. For what values of p does $\lim_{n\to\infty} n^p$ converge? Here, p can be positive, negative or zero.

4. For what values of p does $\lim_{n\to\infty} p^n$ converge? Here, p can be positive, negative or zero.

5. Let $a_n = \frac{-1}{2}$, $b_n = 2$ and $c_n = 2n$. List the terms for each of the sequences appearing in the limits below and calculate
$$\lim_{n\to\infty} (a_n)^n \qquad \lim_{n\to\infty} (b_n)^n \qquad \lim_{n\to\infty} (a_nb_n)^n \qquad \lim_{n\to\infty} (a_n)^n \lim_{n\to\infty} (b_n)^n \qquad \lim_{n\to\infty} (a_nc_n)^n \qquad \lim_{n\to\infty} (b_nc_n)^n$$
You can try some of the other combinations of a_n, b_n and c_n for more practice.

6. Let $a_n = (n+10^n)^{\frac{1}{n}}$. Verify the inequality $10 \le a_n \le (2\cdot 10^n)^{\frac{1}{n}}$ and use the Squeeze Theorem to evaluate $\lim_{n\to\infty} a_n$.

7. Evaluate the limit (as $n \to \infty$) for the following sequences:
$$a_n = \frac{\pi^n}{n!} \qquad b_n = \ln\frac{2n+1}{n+4} \qquad c_n = \frac{n}{n+n^3} \qquad d_n = \frac{(-1)^n n^3 + 2^{-n}}{3n^3+4^{-n}} \qquad e_n = \sqrt{n}\ln\left(1+\frac{1}{n}\right)$$

8. Determine whether the following improper integrals converge or diverge. Show all calculations
$$\int_1^\infty \frac{1}{\sqrt{x}}\,dx \qquad \int_1^\infty \frac{1}{x^2}\,dx \qquad \int_1^\infty x^{-0.87}\,dx \qquad \int_1^\infty x^{13}\,dx \qquad \int_1^\infty \frac{1}{x}\,dx$$

9. In general, for what values of p does the integral $\int_1^\infty \frac{1}{x^p}\,dx$ converge?

10. For what values of a does the integral $\int_1^\infty \frac{1}{x(\ln x)^a}\,dx$ converge?

11. For what values of a does the integral $\int_1^\infty \frac{1}{x^a \ln x}\,dx$ converge?