

Math 354:H1 extra practice for midterm 2

These practice problems are intended to supplement the strongly recommended problem list. The list of topics covered by the exam is not limited to those addressed below.

1. Complete the following statement: If $\mathbf{x}_0$ and $\mathbf{w}_0$ are feasible solutions to the primal and dual problems (as they are written on p. 172 of the text), respectively, and if $\mathbf{c}^T \mathbf{x}_0 = \mathbf{b}^T \mathbf{w}_0$, then ...
(This is theorem 3.6 in chapter 3.2, but don't look it up until you've thought about and completed the statement yourself!"

2. Suppose that $\mathbf{x}_0$ and $\mathbf{w}_0$ are feasible, not necessarily optimal solutions of the primal and dual LPPs, and that the value of the primal objective function at $\mathbf{x}_0$ equals 5. What follows about the value of the dual objective function at $\mathbf{w}_0$?

3. Consider the LPP in chapter 3.6 exercise 1, whose final simplex tableau is printed in the text. Without solving the dual problem by the simplex, two-phase, or dual simplex methods, instead just looking at the final tableau for the given LPP, find the optimal solution of the dual problem.

4. Consider the LPP in chapter 3.6 exercise 1 again, but with $z = 3x_1 + 2x_2 + x_3 + x_4$ as the objective function instead of $x_1 + \dots$. Assume that $x_1 = 3, x_2 = 2, x_3 = 0, x_4 = 0$ is the optimal solution. Formulate the dual LPP, and use complementary slackness to find the values of *all* the dual variables at the optimal solution of the dual problem.

5. Recall our old friend the logging problem, referred to many times in class,
Maximize $z = 40x_1 + 70x_2$ subject to

$$\begin{aligned} x_1 + x_2 &\leq 100 \\ 10x_1 + 50x_2 &\leq 4000 \\ x_1, x_2 &\geq 0 \end{aligned}$$

whose final simplex tableau was

		40	70	0	0	
		x_1	x_2	x_3	x_4	
40	x_1	1	0	$\frac{5}{4}$	$-\frac{1}{40}$	25
70	x_2	0	1	$-\frac{1}{4}$	$\frac{1}{40}$	75
		0	0	$\frac{65}{2}$	$\frac{3}{4}$	6250

Within what range can we change the coefficients $c_1 = 40$ and $c_2 = 70$ without changing the optimal solution?

Within what range can we change the values $b_1 = 100$ and $b_2 = 4000$ so that x_1 and x_2 are still basic variables in the optimal solution?

What will be the optimal solution if c_2 is changed to 30? Do NOT start the whole problem over from the beginning, use the techniques of chapter 3.6.

What will be the optimal solution if b_2 is changed to 4800? Do NOT start the whole problem over from the beginning, use the techniques of chapter 3.6.