ROBERT W. THOMASON
1952–1995

Charles A. Weibel

Like many of his colleagues, Bob Thomason hated to waste energy on trivial matters, like fashion. He made the decision early in life to dress only in black clothing, thus simplifying that portion of his life. With his pointed goatee, he looked like a beat poet to outsiders, but mathematicians knew him as one of the greatest talents of his generation. Few have had the simultaneous grasp of topology, algebraic geometry and K-theory that Thomason did.

Bob had diabetes and always had to strictly control what he ate. This made going to restaurants with Bob an awkward affair, because he would not eat something until he was sure it had no nutritional content. Early in November 1995, just before his 43^{rd} birthday, he went into diabetic shock and died in his apartment in Paris. We are all saddened by his passing.

Here is an overview of Thomason's career. For simplicity, I have focussed upon what I think are his three major results. A retrospective article [W] will appear elsewhere, describing some of his mathematical contributions in more detail.

Robert Wayne Thomason was born in Tulsa, Oklahoma on November 5, 1952. Attracted to Michigan State University by a flexible undergraduate Honors Mathematics program, he spent two years there (1971–73). During his second year at MSU, he published his first paper [T0], in point-set topology. He then spent 1973–1977 as a graduate student in the Princeton University mathematics department, writing his Ph.D. dissertation [T-th] [T1] under the direction of John Moore.

His thesis describes and analyzes a simple but fundamental construction in category theory: the "canonical cofibered category" associated to any diagram D of (small) categories. Since the geometric realization of a small category is a topological space, we obtain a corresponding diagram $|D|$ of topological spaces. The main result in his thesis is that the geometric realization of the canonical cofibered category of D is the homotopy colimit of the diagram $|D|$ of spaces. Because of the elegance and thoroughness of his analysis, this construction has become a basic tool used routinely by topologists.

As he was graduating in June 1977, Thomason discovered the first of his major results: a proof that all infinite loop space machines produce equivalent output. In order to straighten out the technical details of his insight, he immediately enlisted the aid of J. Peter May. In a collaboration May recalls as "delightful interaction," they reduced Bob's argument to a characterization of infinite loop space machines

Author partially supported by NSF grant DMS95-00791

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

by just one axiom: the "group completion" axiom; see [MT]. Thomason's subsequent paper [T2] is a variation on this theme, showing that all one-fold delooping machines also produce equivalent output.

Thomason then went to M.I.T. as a Moore Instructor (1977–1979). During this time, he developed the ideas in his thesis into a series of papers ([LTW], [T3–T6]) in which he studied the homotopy theory of categories, especially symmetric monoidal categories. For example, in [T3] he proves the reassuring result that the abstract homotopy theory of categories does not depend upon a need to pass to geometric realizations, because fibrations, cofibrations, etc. of categories exist as part of a "closed model structure." In [T4] and [T5] he constructed mapping cones, mapping cylinders and other homotopy colimits within the category of small symmetric monoidal categories, and showed that infinite loop space machines send these constructions to the appropriate homotopy colimits of spectra. In a paper humorously entitled "*Beware ...*" [T6], he exposed a subtle but lethal flaw in a putative construction for the ring structure on the K-groups of commutative rings.

In 1979, Thomason went to the University of Chicago to begin a 3-year appointment as a Dickson Assistant Professor. There he developed the notion of cohomological descent for spectra, paralleling the notion of hypercohomology in homological algebra. Today his descent machinery is a basic tool in algebraic K-theory. He also began a 4-year effort to settle the Quillen-Lichtenbaum conjectures, which connect algebraic K-theory to étale cohomology. After the proof of an early partial result collapsed in 1980, Thomason began to feel uncomfortable about the skepticism expressed by others. Perceiving this as persecution, he resigned from his position at Chicago in June 1980.

For the next two years Thomason held an irregular appointment at M.I.T., and then spent a year as a Member at the Institute for Advanced Study. During this period, he finished his opus [T-EC] on the Quillen-Lichtenbaum Conjecture. The papers [T7–T10] and the 4-author paper [DFST] with Dwyer, Friedlander and Snaith, were part of this effort. He also gave two nice applications of [T-EC]: a proof of $\mathbb{Q}_\ell$-adic cohomological purity in [T11], and a Rigidity Theorem with Gillet in [GT] (their proof was contemporaneous with Gabber's).

Thomason's opus [T-EC] contains his second major result. Roughly, it states that the groups K_n can be calculated in terms of étale cohomology for large n, using a formula due to Dwyer and Friedlander. This established the first half of the Quillen-Lichtenbaum Conjecture. The final part of the conjecture, which pins down the values of n, is currently the focus of intensive work in Motivic Cohomology.

In 1983, Thomason joined the mathematics faculty of Johns Hopkins University in Baltimore, where he stayed for six years. During this time, he supervised two Ph.D. dissertations, by Masana Harada [H] and Dongyuan Yao [Y]. In 1983–86, he wrote a series of papers [T12–T16] about equivariant algebraic K-theory.

Starting in 1985, he mounted a sustained 3–year attack upon the problems left over from Grothendieck's opus [SGA6], especially an analysis of how the K-theory of a scheme depends upon its derived category of vector bundles, and how to describe the effect of localization upon K-theory. His successful solution of this problem in 1988 [TT] forms his third major result. In recognition of the importance of his work in [T-EC] and [TT], Thomason was chosen to give an address at the Kyoto International Congress of Mathematicians in 1990 [T20]. His later papers [T22–

T24] are a continuation of [TT] in the spirit of [SGA6].

The story of this 3–year attack reveals much about Thomason's methods. The first step in this program, which he discovered in 1985, was a proof of "cofinality" for Waldhausen K-theory. Today his cofinality theorem is viewed as one of the fundamental results in K-theory. He was awarded a Sloan Fellowship during 1985–87, and spent the entire calendar year 1987 at Rutgers University. While at Rutgers, he put everything into place except for one step: extending perfect complexes from an open subscheme to the entire scheme. On January 22, 1988, he had a dream in which his recently deceased friend Thomas Trobaugh told him how to solve the final step: use "the direct limit characterization of perfect complexes." Awakening with a start, he worked out the argument for the missing step. In gratitude, he listed his friend as a coauthor of the resulting paper [TT].

In October 1989, Thomason made what turned out to be his final career move, to Paris. He accepted a position in the C.N.R.S., attached to Max Karoubi's laboratory URA 212 at the University of Paris 7. While there, he helped Karoubi, Kahn and Kassel run the monthly Paris K-theory seminar, and wrote [T21–T26]. He remained in this position in Paris until his untimely death last November.

References

[DFST] W. Dwyer, E. Friedlander, V. Snaith and R.W. Thomason, *Algebraic K-theory eventually surjects onto topological K-theory*, Invent. Math. **66** (1982), 481–491, MR 84i:14015.

[GT] H. Gillet and R.W. Thomason, *The K-theory of strict hensel local rings and a theorem of Suslin*, J. Pure Appl. Alg. **34** (1984), 241–254, MR 86e:18014.

[H] M. Harada, *A proof of the Riemann-Roch theorem*, Ph.D. thesis, Johns Hopkins University, Baltimore, 1987.

[LTW] D. Latch, R.W. Thomason and S. Wilson, *Simplicial sets from categories*, Math. Zeit. **164** (1979), 195–214, MR 80e:55012.

[MT] J.P. May and R.W. Thomason, *The uniqueness of infinite loop space machines*, Topology **17** (1978), 205–224, MR 80g:55015.

[SGA6] P. Berthelot, A. Grothendieck and L. Illusie, *Théorie des Intersections et Théorème de Riemann-Roch (SGA 6)*, Lecture Notes in Math., vol. 225, Springer-Verlag, 1971, MR 50#7133.

[T0] R.W. Thomason, *A note on spaces with normal product with some compact space*, Proc. AMS **44** (1974), 509–510, MR 51#4165.

[T-th] R.W. Thomason, *Homotopy colimits in* **Cat***, with applications to algebraic K-theory and loop space theory*, Ph.D. thesis, 124 pages, Princeton University, 1977, available from University Microfilms, Ann Arbor, MI 48104.

[T1] R.W. Thomason, *Homotopy colimits in the category of small categories*, Math. Proc. Cambridge Philos. Soc. **85** (1979), 91–109, MR 80b:18015.

[T2] R.W. Thomason, *Uniqueness of delooping machines*, Duke Math. J. **46** (1979), 217–252, MR 80e:55013.

[T3] R.W. Thomason, **Cat** *as a closed model category*, Cahiers Top. Geom. Diff. **21** (1980), 305–324, MR 82b:18005.

[T4] R.W. Thomason, *First quadrant spectral sequences in algebraic K-theory*, Lecture Notes in Math., vol. 763, Springer-Verlag, 1979, pp. 332–355, MR 81c:18018.

[T5] R.W. Thomason, *First quadrant spectral sequences in algebraic K-theory via homotopy colimits*, Comm. Algebra **10** (1982), 1589–1668, 83k:18006.

[T6] R.W. Thomason, *Beware the phony multiplication on Quillen's $\mathcal{A}^{-1}\mathcal{A}$*, Proc. AMS **80** (1980), 569–573, MR 81k:18010.

[T-EC] R.W. Thomason, *Algebraic K-theory and étale cohomology*, Ann. Scient. Éc. Norm. Sup. **18**, 4^e série (1985), 437–552; erratum **22** (1989), 675–677, MR 87k:14016; errata 91j:14013.

[T7] R.W. Thomason, *The Lichtenbaum-Quillen conjecture for $K/\ell_*[\beta^{-1}]$*, Current Trends in Algebraic Topology, CMS Conf. Proc. vol. 2, Part 1, 1982, pp. 117–139, MR 84f:18024.

[T8] R.W. Thomason, *Riemann-Roch for algebraic versus topological K-theory*, J. Pure Appl. Alg **27** (1983), 87–109, MR 85c:14013.

[T9] R.W. Thomason, *Bott stability in algebraic K-theory*, AMS Contemp. Math., vol. 55, 1986, pp. 389–406, MR 87m:18022.

[T10] R.W. Thomason, *The homotopy limit problem*, AMS Contemp. Math., vol. 19, 1983, pp. 407–419, MR 84j:18012.

[T11] R.W. Thomason, *Absolute cohomological purity*, Bull. Soc. Math. France **112** (1984), 397–406, MR 87e:14018.

[T12] R.W. Thomason, *Algebraic K-theory of group scheme actions*, Annals of Math. Study **113** (1987), Princeton U. Press, 539–563, MR 89c:18016.

[T13] R.W. Thomason, *Lefschetz-Riemann-Roch theorem and coherent trace formula*, Invent. Math. **85** (1986), 515–543, MR 87j:14028.

[T14] R.W. Thomason, *Comparison of equivariant algebraic and topological K-theory*, Duke Math. J. **53** (1986), 795–825, MR 88h:18011.

[T15] R.W. Thomason, *Equivariant resolution, linearization and Hilbert's fourteenth problem over arbitrary base schemes*, Adv. Math. **65** (1987), 16–34, MR 88g:14045.

[T16] R.W. Thomason, *Equivariant algebraic vs. topological K-homology Atiyah-Segal-style*, Duke Math. J. **56** (1988), 589–636, MR 89f:14015.

[T17] R.W. Thomason, *The finite stable homotopy type of some topoi*, J. Pure Appl. Alg. **47** (1987), 89–104, MR 88k:14010.

[T18] R.W. Thomason, *A finiteness condition equivalent to the Tate conjecture over $\mathbb{F}_q$*, AMS Contemp. Math., vol. 83, 1989, pp. 385–392, MR 90b:14029.

[T19] R.W. Thomason, *Survey of algebraic vs. étale topological K-theory*, AMS Contemp. Math., vol. 83, 1989, pp. 393–443, MR 90h:14029.

[T20] R.W. Thomason, *The local to global principle in algebraic K-theory*, Proc. International Congress of Mathematicians, Kyoto 1990, vol. 1, Springer-Verlag, 1991, pp. 381–394, MR 93e:19010.

[T21] R.W. Thomason, *Une formule de Lefschetz en K-théorie équivariante algébrique*, Duke Math. J. **68** (1992), 447–462, MR 93m:19007.

[T22] R.W. Thomason, *Le principe de scindage et l'inexistence de K-théorie de Milnor globale*, Topology **31** (1992), 571–588, MR 93j:19005.

[T23] R.W. Thomason, *Les K-groupes d'un schéma éclaté et une formule d'intersection excédentaire*, Invent. Math. **112** (1993), 195–215, MR 93k:19005.

[T24] R.W. Thomason, *Les K-groupes d'un fibré projectif*, Algebraic K-theory and algebraic topology, NATO ASI Series C, vol. 407, Kluwer, 1993, pp. 243–248.

[T25] R.W. Thomason, *The classification of triangulated subcategories*, preprint (1995), Compositio Math. (1996), to appear.

[T26] R.W. Thomason, *Symmetric monoidal categories model all connective spectra*, Theory Appl. Categories **1** (1995), 78–118, (electronic journal http://www.tac.mta.ca/tac/).

[TT] R.W. Thomason and T. Trobaugh, *Higher algebraic K-theory of schemes and of derived categories*, The Grothendieck Festschrift III, Progress in Math., vol. 88, Birkhäuser, 1990, pp. 247–435, MR 92f:19001.

[TT1] R.W. Thomason and T. Trobaugh, *Le théorème de localisation en K-théorie algébrique*, C.R. Acad. Sci. (Paris) **307** (1988), 829–831, MR 89m:18015.

[TW] R.W. Thomason and S. Wilson, *Hopf rings in the bar spectral sequence*, Quart. J. Math Oxford **31** (1980), 507–511, MR 82f:55030.

[W] C. Weibel, *The Work of Robert Wayne Thomason*, preprint (1996).

[Y] D. Yao, *Higher algebraic K-theory of admissible abelian categories and localization theorems*, Ph.D. thesis, Johns Hopkins University, Baltimore, 1990.

Math. Dept., Rutgers University, New Brunswick, N.J. 08903 USA
E-mail address: weibel@math.rutgers.edu