

Numbers refer to problems in *The Arithmetic of Elliptic Curves* (second edition) by J. Silverman

Morphisms, function fields, cocycles

1. 1.3
2. 1.6
3. 1.12
4. Let C be the elliptic curve $y^2z = x^3 - xz^2$ (assume we are working over a field K of odd characteristic). Show that $\sigma([x, y, z]) = [-x, iy, z]$ (where $i^2 = -1$) is an isomorphism of C with itself which fixes $P = [0, 1, 0]$ and is defined over $K(i)$. Show that the rational function x/y is a uniformizer at P and that z/y has order 3 at P . Use this to compute the vector space $L(n)$ of rational functions which are regular on C at all points different from P and have poles of order at most n at P . Show that any isomorphism $\phi : C \rightarrow C$ which preserves P maps each $L(n)$ to itself. Use this to find all isomorphisms of C with itself which map P to P .