FIELDS OF LOW DEGREE n AND SMALLEST DISCRIMINANTS d

This sheet contains all fields with discriminant ≤ 500, as well as the fields of lowest discriminant of degree $n = r_1 + 2r_2 \leq 5$,

Quadratic fields of discriminant ≤ 500	
$r_1 = 0, r_2 = 1$	$r_1 = 2, r_2 = 0$
$-3, -4, -7, -8, -11, -15, -19, -20, -23,$ $-24, -31, -35, -39, -40, -43, \ldots$	$5, 8, 12, 13, 17, 21, 24, 28, 29, 33, 37,$ $40, 41, 44, 53, 56, 57, 60, 61, 65, \ldots$
$\ldots, -479, -483, -487, -488, -491, -499$	$\ldots, 481, 485, 488, 489, 492, 493, 497$

Cubic fields of discriminant ≤ 500					
$r_1 = 1, r_2 = 1$				$r_1 = 3, r_2 = 0$	
-23	$x^3 - x^2 + 1$	-268	$x^3 - x^2 - 3x + 5$	49	$x^3 - x^2 - 2x + 1$
-31	$x^3 + x - 1$	-283	$x^3 + 4x - 1$	81	$x^3 - 3x - 1$
-44	$x^3 - x^2 + x + 1$	-300	$x^3 - x^2 - 3x - 3$	148	$x^3 - x^2 - 3x + 1$
-59	$x^3 + 2x - 1$	-307	$x^3 - x^2 + 3x + 2$	169	$x^3 - x^2 - 4x - 1$
-76	$x^3 - 2x - 2$	-324	$x^3 - 3x - 4$	229	$x^3 - 4x - 1$
-83	$x^3 - x^2 + x - 2$	-327	$x^3 - x^2 - 2x - 3$	257	$x^3 - x^2 - 4x + 3$
-87	$x^3 - x^2 + 2x + 1$	-331	$x^3 - x^2 + 3x - 4$	316	$x^3 - x^2 - 4x + 2$
-104	$x^3 - x - 2$	-335	$x^3 - x^2 + 4x + 1$	321	$x^3 - x^2 - 4x + 1$
-107	$x^3 - x^2 + 3x - 2$	-339	$x^3 - x^2 - x + 4$	361	$x^3 - x^2 - 6x + 7$
-108	$x^3 - 2$	-351	$x^3 + 3x - 3$	404	$x^3 - x^2 - 5x - 1$
-116	$x^3 - x^2 - 2$	-356	$x^3 - x^2 + x + 7$	469	$x^3 - x^2 - 5x + 4$
-135	$x^3 + 3x - 1$	-364	$x^3 + 4x - 2$	473	$x^3 - 5x - 1$
-139	$x^3 - x^2 + x + 2$	-367	$x^3 - x^2 + 2x + 3$		
-140	$x^3 + 2x - 2$	-379	$x^3 - x^2 + x - 4$		
-152	$x^3 - x^2 - 2x - 2$	-411	$x^3 - x^2 + 5x - 2$		
-172	$x^3 - x^2 - x + 3$	-419	$x^3 - 4x - 5$		
-175	$x^3 - x^2 + 2x - 3$	-424	$x^3 - x^2 + 8$		
-199	$x^3 - x^2 + 4x - 1$	-431	$x^3 - x - 8$		
-200	$x^3 - x^2 + 2x + 2$	-436	$x^3 + x - 4$		
-204	$x^3 - x^2 + x - 3$	-439	$x^3 - x^2 - 2x + 5$		
-211	$x^3 - 2x - 3$	-440	$x^3 + 2x - 8$		
-212	$x^3 - x^2 + 4x - 2$	-451	$x^3 - x^2 - 5x + 8$		
-216	$x^3 + 3x - 2$	-459	$x^3 - 6x - 7$		
-231	$x^3 - x^2 + 3$	-460	$x^3 - x^2 + 5x - 3$		
-239	$x^3 - x - 3$	-472	$x^3 - 5x - 6$		
-243	$x^3 - 3$	-484	$x^3 - x^2 + 4x + 2$		
-244	$x^3 - x^2 - 4x + 6$	-491	$x^3 - x^2 + x + 4$		
-247	$x^3 + x - 3$	-492	$x^3 - x^2 + 3x + 3$		
-255	$x^3 - x^2 - 3$	-499	$x^3 + 4x - 3$		

Note: The fields given by $x^3 - 6$ and $x^3 - 12$ are nonisomorphic and have the same discriminant -972. This is the lowest discriminant absolute value for which this happens.

Quartic fields of discriminant ≤ 500 or minimal discriminant for fixed r_1, r_2 Quartic fields with no quadratic subfields		
$r_1 = 0, r_2 = 2$	$r_1 = 2, r_2 = 1$	$r_1 = 4, r_2 = 0$
$229\ x^4 - x + 1$	$-283\ x^4 - x - 1$	$1957\ x^4 - 4x^2 - x + 1$
$257\ x^4 + x^2 - x + 1$	$-331\ x^4 - x^3 + x^2 + x - 1$	$2777\ x^4 - x^3 - 4x^2 + x + 2$
	$-491\ x^4 - x^3 - x^2 + 3x - 1$	
Quartic fields with quadratic subfields		
$117\ x^4 - x^3 - x^2 + x + 1$	$-275\ x^4 - x^3 + 2x - 1$	$725\ \ x^4 - x^3 - 3x^2 + x + 1$
$125\ x^4 - x^3 + x^2 - x + 1$	$-400\ x^4 - x^2 - 1$	$1125\ x^4 - x^3 - 4x^2 + 4x + 1$
$144\ x^4 - x^2 + 1$	$-448\ x^4 - 2x^3 + x^2 + 2x - 1$	$1600\ x^4 - 6x^2 + 4$
$189\ x^4 - x^3 + 2x + 1$	$-475\ x^4 - 2x^3 + 2x^2 - x - 1$	
$225\ x^4 - x^3 + 2x^2 + x + 1$		
$256\ x^4 + 1$		
$272\ x^4 + x^2 - 2x + 1$		
$320\ x^4 - 2x^3 + 2x + 1$		
$333\ x^4 - x^3 - 2x^2 + 3$		
$392\ x^4 - x^3 + x + 1$		
$400\ x^4 + 3x^2 + 1$		
$432\ x^4 + 3$		
$441\ x^4 - x^3 - x^2 - 2x + 4$		

Note : The fields given by $x^4 - 2x^3 + 2x^2 - 2$ and $x^4 + 6x^2 - 23$ both have discriminant -1472 and $r_1 = 2, r_2 = 1$. The second has a quadratic subfield, while the first does not. This is the lowest absolute value of a discriminant of a quartic which is the discriminant of nonisomorphic fields.

Quintic fields of minimal discriminant for fixed r_1, r_2		
$r_1 = 1, r_2 = 2$	$r_1 = 3, r_2 = 1$	$r_1 = 5, r_2 = 0$
$1609\, x^5 - x^3 - x^2 + x + 1$	$-4511\, x^5 - x^3 - 2x^2 + 1$	$14641\, x^5 - x^4 - 4x^3 + 3x^2 + 3x - 1$
$1649\, x^5 - x^4 + x^3 - x + 1$	$-4903\, x^5 - x^4 - x^3 + 2x^2 - x - 1$	$24217\, x^5 - 5x^3 - x^2 + 3x + 1$

References:

n=3 Delone and Fadeev, Acad. Sci. USSR, Inst. Math. Steklov, 1940 page 160

n=4 Godwin, Quart. J. Math. Oxford (8) 957, 214–222

n=5 Hunter, Proc. Glasgow Math Assoc. III, 1957, pg. 57

Remark: The references above explain the method of finding such fields. The fields listed are not in the same form as those in the references, as the defining equations have been chosen small in the sense that the sum of the squares of the absolute values of the roots is small.