

Week 8 workshop problems

1. Show that if $u(x, y)$ is harmonic and non-constant on a connected open set Ω in \mathbf{R}^2 then $u(\Omega)$ is an open interval of the real line.
2. Show that the Schwarz lemma can be generalized to the situation that $f(z)$ is a holomorphic map of an open disk centered at the origin of radius R to an open disk centered at the origin of radius M such that $f(z_0) = w_0$. Show that

$$\left| \frac{M(f(z) - w_0)}{M^2 - \bar{w}_0 f(z)} \right| \leq \left| \frac{R(z - z_0)}{R^2 - \bar{z}_0 z} \right|$$

3. Show that the Schwarz lemma variant of problem 2. above implies Liouville's theorem by letting z approach z_0 and R approach infinity.