Mathematics 503 Complex Analysis Fall 2017

Week 7 workshop problems

1. Show that the function $u(x, y) = e^x(x \cos y - y \sin y)$ is harmonic on the plane by finding an entire function which has real part $u(x, y)$.

2. Let U be the complex plane with all nonpositive real numbers deleted. Show that $u(x, y) = \sqrt{(x + \sqrt{x^2 + y^2})/2}$ is harmonic on U by finding a holomorphic function on U which has real part $u(x, y)$.

3. Show that the tangent function maps the strip $\{x + iy | -\pi/2 < x < \pi/2\}$ conformally onto its image and has inverse

$$h(z) = \frac{1}{2i} Log \frac{1 + iz}{1 - iz}.$$

Hint: write the tangent as a composition of conformal maps which are made visible by the formula $\tan z = i(1 - e^{2iz})/(1 + e^{2iz})$